Measuring Social Mobility

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Measuring Social Mobility*

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1. *Social Mobility as Shown in a Transition Matrix*

A conclusion which clearly emerges from the series of investigations recently published under the title *Social Mobility in Britain* (edited by D. V. Glass, 1954) is that while the concept of social mobility is complex its study is best furthered by the kind of quantitative approach so widely adopted there. This, again, is in accord with views expressed in earlier investigations such as that of Ginsberg (1929). Perhaps the most interesting table of statistics resulting from such quantitive studies is one setting out the relation between the social statuses of fathers and their sons as derived from a series of interviews of a sample of the population. An example of such a table is given below as Table 1 which is taken from the volume edited by Glass; it is based on the results of a random sample of some 3,500 males, resident in England and Wales and aged 18 years and over, interviewed by the Social Survey in 1949.

In this paper such a table will be termed a transition matrix, for the coefficients of such a matrix will be regarded as giving the probability of a family's transition from one social status to another. With the help of the assumption that these probabilities are constant over time,† a number of new summary measures will be derived which seem to have advantages over those currently used in assessing the degree of social mobility.‡

This note is restricted to the setting out of the method of analysis and to its application to the main table just mentioned. The many qualifications to which the figures in this table are subject have been amply set out by Glass and Hall and there is no need to repeat them here; as far as I can see they do not affect the interpretations suggested in this paper to any substantial extent. A point worth bearing in mind, however, in interpreting the calculation given below is that "social class" has been treated as if it related only to the male side of the family line; this is largely because social class has in these studies been measured by occupation. The only further assumption made in the present analysis is that the influence of one's ancestors in determining one's class is transmitted entirely through one's father; so that if the influence of one's father has been taken into account, then the total influence of one's ancestors is accounted for. A more detailed analysis, which explicitly brought in any additional and separate effects of earlier generations and included both sides of the family, would seem to be feasible in principle, though it may well become rather complicated. No attempt at any such generalization is made in this paper.

2. *The Equilibrium Distribution of the Social Classes*

It is convenient to begin by setting out the transition matrix used as the basis for all the subsequent calculations in this paper. This is done in Table 1.§ The j^{th} element of i^{th} row of this matrix, to be denoted by p_{ij}, gives the proportion of fathers in the j^{th} social class whose sons move into the i^{th} social class; alternatively, if it is supposed that there is some unambiguous

* In preparing this paper I have considerably benefitted from discussion with Richard Stone, Director of the Department of Applied Economics, with M. J. Farrell, and with my colleagues at the Department.

† Some evidence on this is given by Glass and Hall in one of the central papers appearing in—Glass (1954), pp. 185–8.

‡ The basis of the approach outlined here is the theory of Markov chains which has applications in a number of fields in the natural sciences. A good account may be found in the work by Feller (1950), especially chapter 15.

§ Taken from Glass and Hall, op. cit., p. 183. The matrix as given here is the transpose of that published by Glass and Hall to conform with the algebraic treatment below.

method of tracing the family line through time, then p_{ij} represents the probability of a transition by a family from class j into class i in the interval of one generation. In this example

$$i, j = 1, 2, \dots 7,$$

and a brief description is given at the side of the table of the classes distinguished in this study which are largely on an occupational basis.

TABLE 1

The Social Transition Matrix in England, 1949

[The j^{th} element in the i^{th} row of this table gives the proportion of fathers in the j^{th} social class whose sons are in the i^{th} social class.]

Class	1	2	3	4	5	6	7
1. Professional and high administrative	0·388	0·107	0·035	0·021	0·009	0·000	0·000
2. Managerial and executive	0·146	0·267	0·101	0·039	0·024	0·013	0·008
3. Higher grade supervisory and non-manual	0·202	0·227	0·188	0·112	0·075	0·041	0·036
4. Lower grade supervisory and non-manual	0·062	0·120	0·191	0·212	0·123	0·088	0·083
5. Skilled manual and routine non-manual	0·140	0·206	0·357	0·430	0·473	0·391	0·364
6. Semi-skilled manual	0·047	0·053	0·067	0·124	0·171	0·312	0·235
7. Unskilled manual	0·015	0·020	0·061	0·062	0·125	0·155	0·274

If the table is taken a row at a time, the elements show how the probabilities of entering a given class vary with the class of one's father. Since everyone must be in some class (whatever the class of one's father) it follows that the sum of each column is unity.

Knowing the proportions of fathers in each social group at any time it is possible to derive in an obvious way with the help of this matrix the distribution of the sons among the various social classes. Thus, let s_{it} denote the proportion in the i^{th} social class at time t, then

$$s_{i(t+1)} = \Sigma\, p_{ij}\, s_{jt}; \qquad (1)$$

alternatively, in the more convenient notation of matrix algebra, let s_t denote the (column) vector of the proportions in the various classes at time t, and let P be the transition matrix, then

$$s_{t+1} = P\, s_t. \qquad (2)$$

The unit of time implied in this equation is a generation.

An obvious question now arises: supposing that the matrix P remains unchanged over time, what distribution of the population among the various social classes can be expected at any subsequent period, say, n generations later? Applying the relationship (2) successively, there results

$$s_{t+2} = Ps_{t+1} = P(Ps_t) = P^2 s_t,$$

$$s_{t+3} = Ps_{t+2} = P^3 s_t,$$

and, in general,

$$s_{t+n} = P^n s_t. \qquad (3)$$

This shows the distribution at time $(t + n)$ as dependent on the distribution at time t and on the transition matrix P raised to the n^{th} power.

The interest of the development (3) lies not so much in the detailed relationships for any given period of time but rather in the following general mathematical result which is here stated without proof.* *For any matrix of the kind* P, *the distribution of the social classes tends in time to a value,* $\bar{s}$, *which is independent of the original distribution.* The distribution $\bar{s}$ is here called the equilibrium distribution; it satisfies the equation

$$\bar{s} = P\,\bar{s} \qquad (4)$$

* A proof of this theorem for a general stochastic matrix will be found in Feller (1950), p. 325.

and once this distribution is reached it will be maintained through time. The equilibrium distribution so defined thus depends only on the structural propensities of the society and not on the distribution of the population among the classes found at any instant.

The equilibrium distribution is also independent of the unit of time in which the elements of P are measured; suppose, for example, that observations were taken to allow the writing of an equation similar to (1) showing the relationship between the social statuses of grandson and grandfather. Every element of the transition matrix would then be different since it would refer to a transition during a period of two generations instead of one generation. The equilibrium distribution corresponding to such a matrix would however be unchanged. For, if the matrix relating the statuses of sons to fathers is P, that relating those of grandsons to grandfathers will be P^2 (provided, of course, that nothing has happened to change the characteristics of the society in the period considered), and when these matrices are raised to the n^{th} power they obviously tend to the same value as n tends to infinity.

TABLE 2

Actual and Equilibrium Distributions of the Social Classes in England

Class	*Actual Distributions of* *Fathers* (1)	*Sons* (2)	*Equilibrium Distribution* (3)	*Column* (3) / *Column* (2) (4)
1. Professional	0·037	0·029	0·023	0·79
2. Managerial	0·043	0·046	0·042	0·91
3. Higher grade non-manual	0·098	0·094	0·088	0·94
4. Lower grade non-manual	0·148	0·131	0·127	0·97
5. Skilled manual	0·432	0·409	0·409	1·00
6. Semi-skilled manual	0·131	0·170	0·182	1·07
7. Unskilled manual	0·111	0·121	0·129	1·07

Table 2 gives the actual distribution of the classes, as estimated from the sample of observations, for the sons and for the fathers, together with the equilibrium solution. On the whole it will be seen that the distributions are remarkably similar, and hence that even if the transition matrix of Table 1 continues to hold entirely unchanged over the next few generations, no great changes are to be expected in the distribution of the population among the various social classes.

A more precise comparison of the results is made in the last column of the above table which shows the ratio of the proportion of the current generation, that is of the sons, in each class to the proportion to be expected in equilibrium. These figures are very regular and indicate that if recruitment to the various classes proceeds as it has in the past generation then the proportion in the uppermost class will decline by about a fifth, while the proportions in the two lowest classes will rise somewhat.

Since the definition of social class which was used in compiling these figures is based largely on occupation there are obvious qualifications to the validity of the last conclusions, though these would not seem to detract from the interest of this kind of calculation. However, my friend Mr. M. J. Farrell has suggested to me that when there are considerable differences between the observed distributions of fathers and sons it is more reasonable to suppose that these are for the greater part due to once-for-all shifts in the distribution of the various kinds of occupations made available by the industrial system. When this is so it is desirable to make what amendments one can to the observed transition matrix before proceeding with the calculations suggested in the following sections. A method of making such adjustments is suggested in Appendix A, and the same formal method can be used for correcting the matrix for differences in the net reproduction rates of the various classes. Since in the case examined here the differences between the distributions of the two observed generations are not very great, adjustments of this sort are not considered in the main text.

3. *The Average Time Spent in a Social Class*

In this section the transition matrix is interpreted on the basis of the supposition, mentioned above, that there is an unambiguous method of tracing a family line through time. No fundamental distinction is drawn between intra-generation mobility and inter-generation mobility, but these are both regarded as referring to the same phenomena and are to be described by similar sets of data; the only difference between these concepts is the period of time to which the phenomena relate. Problems arising in the latter connexion can be treated more satisfactorily along the lines to be suggested in the following section.

If there were a complete absence of mobility in a society each family line would stay in its social class for a theoretically infinite period of time; on the other hand, the more mobile the society the shorter the period during which a particular family line would be found in a given social class. The average period spent in a social class, as compared with the period spent in that class in a comparable but perfectly mobile society, would seem to provide a measure (but, of course, not the only measure) of the mobility of a given society.

The average number of generations spent in a social class is most simply calculated as follows. Let there be s_j families in class j in the current generation. Of these the number $s_j p_{jj}$ will be found in the j^{th} class in the next generation; the number $s_j p_{jj}^2$ will be found in the third generation, and so on. Hence the total time spent in the j^{th} class by the s_j families at present in that class is

$$s_j + s_j p_{jj} + s_j p_{jj}^2 + \ldots$$

On dividing by s_j there results the average time, t_j, spent by a family in that class; thus

$$t_j = 1 + p_{jj} + p_{jj}^2 + \ldots$$

that is,

$$t_j = \frac{1}{1 - p_{jj}}. \qquad (5)$$

An alternative approach with the help of which it is possible to obtain also the standard deviation of the time spent in each social class is given in Appendix B. It is there shown that the standard deviation is

$$\sigma_j = \frac{\sqrt{p_{jj}}}{1 - p_{jj}}. \qquad (6)$$

In order to use the measures t_j in assessments of the mobility of a society it is necessary to know what would be the corresponding values of these measures in a society that is perfectly mobile. This must now be defined.

In terms of the transition matrix, a perfectly mobile society is a society in which the probability of entering a particular social class is independent of the class of one's father; so that all the elements in each row of the matrix would be substantially equal (to any given degree of approximation), though there would generally be differences between the rows. A more general definition of perfect mobility would make the probability on entering a class substantially independent of that of one's n^{th} progenitor; where the first progenitor is defined as the father, the second progenitor as the grandfather, and so on. Just how small the value of n, and how small the differences between the elements in any row, should be before the society is judged as adequately mobile are questions of social policy from which a discussion of this sort must abstract as far as possible; some calculations which bear on this are, however, given in the next section.

Of the infinitely many possible perfectly mobile societies, the one chosen here as a basis for comparison with our actual society is that which has the same equilibrium distribution of the social classes. Hence the transition matrix of the perfectly mobile society which is comparable to our own, is composed of the values given in column (3) of Table 2; it will be a matrix which has that column repeated seven times. Since the equilibrium distribution depends on the broadness of the definition of each class, it will be apparent that by making a comparison with it, the measures t_j are standardized for these differences in the broadness of the classes. Values of t_j for different classes therefore become directly comparable.

If there were other information available on long term trends in the occupational distribution of society it may be preferable to use some other distribution in place of the equilibrium distribution suggested here. For example, if it were thought that the present distribution between the classes was likely to continue unchanged over the next few generations (and that if anything had to change it would be the transition matrix that would change in order to conform to such a distribution) then it would be more sensible to use as a basis for comparison a transition matrix of a perfectly mobile society which led to such a distribution. The course adopted here is thought preferable since it makes use of what knowledge there is available (in the matrix) of the latent tendencies of society.*

The results of the calculations based on the formulæ given above are set out in Table 3. The first column shows that even when only seven groups are distinguished in the population, in no case does the average number of generations spent by a family in a particular social class exceed

TABLE 3

The Average Number of Generations Spent in Each Social Class in England To-day and in a Similar but Perfectly Mobile Society

Class	*Average Number of Generations in* *England To-day* (1)	*Mobile Society* (2)	*Column (1) / Column (2)* (3)	*Standard Deviation of Col. (1)* (4)
1. Professional	1·63	1·02	1·59	1·02
2. Managerial	1·36	1·04	1·30	0·71
3. Higher grade non-manual	1·23	1·10	1·12	0·54
4. Lower grade non-manual	1·27	1·15	1·11	0·58
5. Skilled manual	1·90	1·69	1·12	1·30
6. Semi-skilled manual	1·45	1·22	1·19	0·81
7. Unskilled manual	1·38	1·15	1·20	0·72

two. The longest time is 1·9 generations for the skilled manual and routine non-manual workers, which is the broadest of the groups distinguished. The impression that families may spend longer than this in a particular class is no doubt due to the wide dispersion of the actual times†; as will be seen from column (4) of the table, the standard deviations are in fact considerable.

The second column of the table shows the average number of generations that would be spent in each class if the society were perfectly mobile. The least time, 1·02 generations, would be spent in the top group of professional and high administrative workers, which is the smallest group; whereas 1·69 generations is the average time for the broadest group.

The measures of the immobility of the social structure are given in column (3). It will be seen that the bottom five groups are affected approximately equally by the effects of immobility; the net result being that the average family spends between 10 and 20 per cent. more time in these social classes than if society were completely mobile. Values as low as this may be considered as negligible in view of the biases to which these figures may be subject.‡

However, in the top group the effects of self-recruitment are such as to raise the time spent by some 60 per cent. over that in a perfectly mobile society; and this figure may well seem excessive when compared with the others. For the second group from the top the excess time is somewhat lower at 30 per cent.

4. *The Variation of Mobility with Time*

The measure of mobility given in the last section is of a particularly simple sort since it depends only on the amount of self-recruitment in each class—that is, on the diagonal elements of the transition matrix. The other elements only enter into the calculation of the equilibrium distribution which is used as a standardizing factor.

* The alternative calculations given in Appendix A assume that there is no difference between the present and the equilibrium distributions; so that if it were thought that the present distribution will continue to hold these alternative calculations are to be preferred.

† Another reason may be that when thinking in terms of social classes in this connexion, it is not convenient to distinguish as many as seven classes and, of course, the broader the grouping of the classes the longer is the time spent in any class. Thus, if Classes 3, 4 and 5 are grouped into one large "middle class", it is found that the average time spent by a family line in that class is 3·3 generations.

‡ The principal bias is that at the time of questioning the subject may not have reached his final status, so that the full amount of movement between generations is understated.

In this section a set of rather more complicated measures is proposed which have the advantage that it is possible to assess with their aid the amount of movement among the various social classes achieved in successive generations; these measures depend more directly on all the elements of the matrix. They arise from the observation that in societies such as those described by Table 1 not only is the degree of self-recruitment higher than in a perfectly mobile society, but also the degree of recruitment from adjacent classes is higher than that from the more distant classes.

Since it is also true that the probability of moving into adjacent classes is higher than that for movements into the more distant classes, the net result is that if we consider, say, the number of grandfathers whose sons have moved out of their social class but whose grandsons have moved back into that same social class, then this number will also tend to be higher* than that to be found in a perfectly mobile society.

An example may make this clearer. Referring to Table 1 it will be seen that the probability of moving from Class 1 to Class 2 in the first generation, and then back into Class 1 in the second generation, is given by the value $0\cdot146 \times 0\cdot107 = 0\cdot016$. On the other hand, the probability of moving from Class 1 to the more distant Class 5 and back again is given by the much smaller value of $0\cdot140 \times 0\cdot009 = 0\cdot001$. In the similar but perfectly mobile society (based on the equilibrium values given in Table 2) the value of these probabilities would be, respectively, $0\cdot042 \times 0\cdot023 = 0\cdot001$, and $0\cdot409 \times 0\cdot023 = 0\cdot009$. So that for this society the probability of moving into an adjacent class and back is higher than that in a perfectly mobile society, while for a distant class it is smaller.

The net effect—that is, the probability of moving into any other class from the first class and then back into the first class—may be found by summing over the six classes and will be found to have the value $0\cdot025$, as compared with the value of $0\cdot022$ to be found in perfectly mobile society. If the probability of self-recruitment is included in these figures, as is especially convenient when these measures are generalized to n generations, there results the probability of finding one's grandson in one's own class irrespective of whether one's son has remained in the same class or not. The values obtained for the first class in the actual and perfectly mobile society are then $0\cdot176$ and $0\cdot023$. The ratio of these two figures is $7\cdot6$ and may be termed the Immobility Factor for the second generation relating to the first class: it shows that for this class there are nearly eight times as many grandsons in one's own class than there would be in perfectly mobile society.

The generalization to great-grandsons, great-great-grandsons and so on, to yield Immobility Factors for the third generation, fourth generation and so on, is now straightforward. The possibility of doing this is considerably simplified by the following theorem which holds for any perfectly mobile society.

Suppose that the number of family lines in the j^{th} class were enumerated now and, after an interval of n generations, they were enumerated again. At the second enumeration it would be found that a certain proportion, say $m_{jj}^{(n)}$, of those family lines originally in the j^{th} class were still in that class. The theorem states that in a perfectly mobile society this proportion does not depend on the number of intervening generations; that is $m_{jj}^{(n)}$ is independent of n.

The proof of this proposition, if any is needed, is simple. For the proportions $m_{jj}^{(n)}$ are the elements of a matrix M^n defined by an equation analogous to (3). Now, if M is the transition matrix of any perfectly mobile society that is, for all i and k it is true that $m_{ik} = m_{ii}$, then the elements of M^2 are given by

$$\begin{aligned} m_{ij}^{(2)} &= \sum_k m_{ik}\, m_{kj} = \sum_k m_{ii}\, m_{kj} \\ &= m_{ii} \sum_k m_{kj} \\ &= m_{ii} \end{aligned}$$

since the sum of any column is unity. Hence

$$M = M^2 = M^3 = \ldots, \qquad (7)$$

and, in particular, $m_{jj} = m_{jj}^{(n)}$.

On the other hand, in a society that is at all immobile, the ratio of family lines to be found in the j^{th} class now to those who were originally in that class n generations ago (and have either stayed

* In general it will only be different: if the amount of self-recruitment is particularly high the number will be *less* than that in a perfectly mobile society.

in that class for the intervening n generations or have left that class and returned to it) will depend on the value of n (so long as n is finite). The ratios will in fact be given by the diagonal elements, $p_{jj}^{(n)}$, of the matrix P^n defined in equation (3) above.

TABLE 4

Immobility Ratios for the First to the Fourth Generations in England

Class	*First*	*Second*	*Third*	*Fourth*
1. Professional	16·9	7·6	4·0	2·4
2. Managerial	6·4	2·9	1·8	1·4
3. Higher grade non-manual	2·1	1·3	1·1	1·1
4. Lower grade non-manual	1·7	1·1	1·0	1·0
5. Skilled manual	1·2	1·0	1·0	1·0
6. Semi-skilled manual	1·7	1·2	1·1	1·0
7. Unskilled	2·1	1·3	1·1	1·0

The measures of immobility for the t^{th} generation are then defined as $p_{jj}^{(t)}/m_{jj}$. The following points are worth noting.

First, for $t = 1$, this measure has a direct relationship with the measure based on the average time spent in each class which was considered in the last section. However, it is not possible from a knowledge of one of these measures alone to obtain the other.

Secondly, as t becomes larger, the Immobility Ratios for all classes tend to unity, indicating that after a sufficiently long time the number of descendants to be found in one's own class differs arbitrarily little from that to be found in a perfectly mobile society.

Thirdly, the Immobility Ratios for the first generation are almost* identical to the Indexes of Association used in the studies edited by Glass. The ratios for the t^{th} generation proposed here may therefore be regarded as generalization of this measure.

Fourthly—and this is perhaps the main interest of this set of ratios—suppose that on the basis of external considerations a perfectly mobile society was defined as one in which the chance of entering a particular class could depend on one's father's class but would be independent of one's grandfather's class. Then the Immobility Ratio for the second generation would give a correct measure of the deviation of the social structure from this ideal; and the ratios for this and all the subsequent generations would still be valid as measures of the progression of mobility through time. In general, if in the perfectly mobile society the probability of entering a class is defined as being independent of one's t^{th} progenitor, this requires that only the Immobility Ratios for the t^{th} generation and onwards be considered, but the calculations do not have to be modified in any way.

Immobility Ratios for the first four generations have been calculated from the data given in Table 1 and the results are brought together in Table 4. They show that in the first generation there is a fair amount of immobility: the largest index is that for the first group of professionals and high administrators, and the lowest value is that for the fifth group where there is a negligible amount even of first generation immobility.

By the time the third generation is reached all the classes except the upper two may be judged to be fully mobile for practical purposes. The Index shows, however, that the first class has four times as many great-grandsons in its own group than would be found in a perfectly mobile society. Even by the time the fourth generation is reached, the index for this class is still higher than that for the five lower classes in the first generation.

The impression to be gained from this table as to the extent of social immobility, and its importance as a factor in social policy, may at first sight be rather different from that gained from the previous table from which it appeared that, on the average, the time spent in any social class did not exceed two generations. It is, therefore, worth adding that the two sets of measures are entirely consistent. The impression of a greater degree of immobility which may be derived from the calculations of this section is due to the fact that they are not based merely on the average values of the distribution : though Table 4 shows that there are four times as many of one's own descendants to be found in the first class in the third generation than there would be in a perfectly

* The sole difference is that the equilibrium distribution is here used to standardize the ratio instead of the actual distribution of sons used by Glass and others.

mobile society, the absolute numbers involved are very small and this value therefore received only a small weight in calculating the average number of generations spent in that class.

5. *Some Extensions*

The main implications to be drawn from a probability approach to the theory of social mobility have been set out in the previous sections. In this section there are outlined two extensions to the formal framework and some remarks are added on possible further work.

The first extension is an implication of the basic theorem mentioned in section 2 that the equilibrium distribution is independent of any initial distribution, s_t. This theorem implies at once that the probability of being in the j^{th} class n generations after being in the i^{th} class tends to a value, as n increases, which is independent of i; that is

$$p_{ji}^{(n)} \rightarrow \bar{s}_j. \qquad (8)$$

This merely assures us that our formal system has the commonsense property that the probability of being in a particular class is independent of the class of one's ancestor, say, a thousand years ago.

Secondly, any distribution, s_t, may be viewed either as giving the proportions in the various classes in which it is to be expected that a large population will be found at time t; or as giving the probabilities that any particular family line will be found in any of the social classes at that time. From the latter point of view, the equilibrium distribution similarly gives the probabilities that any particular family line will be found in any of the social classes at all times (except possibly for a first few generations).

Since any particular family line will throughout history spend its time wandering from one social class to another, it is possible to compute the average time between its returns to any particular class, or the *mean recurrence times*. These are equal to the reciprocals of the elements, $\bar{s}_j$, of the equilibrium vector.*

Calculations based on the equilibrium values given in Table 2 show that the mean recurrence time varies between 2·4 generations for the fifth class of skilled manual workers† to a maximum of 41·6 generations for the top class of professional workers. It will be obvious from what has been said in earlier sections that these values are entirely determined by the broadness of definition of the various social classes and hence do not directly cast any light on the extent of social mobility. But these measures may be of help as another way of assessing the broadness of the classes.

It would obviously be valuable to carry out comparable computations of the average time spent in each social class (and the other measures proposed here) both for this country at various past times, and for other countries for which comparable data are available. When this is done it will become rather more important to face a problem that has been avoided in this paper as requiring separate treatment: that of the assessment of the statistical errors in the estimation of the transition matrix and in its various transformations.

The problems that arise under this head do not appear to be very serious since the classical formula for the variance of a proportion, $p(1-p)/n$, where n is the number of observations, can be applied in a fairly straightforward way in estimating the variances of the elements of the transition matrix. Some mild problems will, however, have to be solved in obtaining confidence limits for the derived summary measures proposed in this paper.

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* An intuitive proof of this may be found helpful. If the probability of obtaining a 1 with a balanced die is 1/6th, a little reflexion or experimentation should convince the reader that the mean recurrence time between throws of a 1 will be six. Consider next the probability of throwing a 1 or a 2; this will be 1/3rd and on the average it will be necessary to wait for three throws before either a 1 or a 2 appears. Similarly, if $\bar{s}_j$ is the probability of being in class j at any time through history, the mean time between recurrence in j will be $1/\bar{s}_j$ as stated. The rigorous proof is rather more difficult and is given in Feller (1950), pp. 259–61.

† This is presumably the basis of the popular saying "From clogs to clogs—three generations".

APPENDIX A

*A Correction for Shifts in the Occupational Structure**

The differences between the observed distributions of sons and fathers have, in the main part of the text, been attributed entirely to the results of the forces making for social mobility as represented in the transition matrix. The alternative assumption to be investigated here is that, if the effects of social mobility alone were observed, there would be no differences between the proportions of sons and fathers in each class. What differences there are should be ascribed to extraneous factors such as shifts in the occupational structure or differences in the reproduction rates of the classes. This alternative assumption is at the other extreme from that made in the main part of the text and a comparison of the two sets of calculations should provide limits for the true state of affairs.

The observed transition matrix P can then be considered as the resultant of the effects of social mobility, represented in a matrix Q, and the effects of occupational shifts represented in a matrix R, such that

$$P = Q\,R. \qquad (9)$$

A simple assumption to make in estimating the elements of R is to suppose that the numbers in any class after the change in the occupational structure are composed of a weighted average of those in that class and in one of the adjacent classes before the change. The adjacent class should be chosen as the one† above or below so as to make both weights positive. There will generally be a unique way of doing this, and this is illustrated in Table 5 which is constructed as follows.

TABLE 5

Changes in the Occupational Structure of England in the Last Generation

Class	1	2	3	4	5	6	7	$\Sigma\, s_{t+1}$
1	103	—	—	—	—	—	—	103
2	26	133	—	—	—	—	—	159
3	—	17	313	—	—	—	—	330
4	—	—	32	427	—	—	—	459
5	—	—	—	91	1,338	—	—	1,429
6	—	—	—	—	172	421	—	593
7	—	—	—	—	—	37	387	424
$\Sigma\, s_t$	129	150	345	518	1,510	458	387	3,497

The actual distribution of the fathers, based on the sample figures, is written down in the bottom row and the distribution of the sons, which is to be derived from this, is written down in the extreme right-hand column. Since there are 129 fathers in the first class and only 103 sons in that class, it is assumed that the balance of 26 sons has moved into the second class. The number 103 is therefore written in the leading position and the number 26 below it. The remaining 133 sons, which are required in the second class to give the observed total of 159, are then taken from the 150 fathers in that class so leaving a balance of 17 to be carried forward to the third-class. This procedure is continued till the final class is reached which balances out exactly, since the number of sons and fathers is equal in total.

The matrix R is then derived by dividing each column by the sum of the elements in it, and this matrix will then satisfy the equation.

$$s_{t+1} = R s_t. \qquad (10)$$

Of course, if there were any more direct information available on the way any particular factor affected recruitment to the various classes, it should be incorporated into the definition of R.

* I owe the substance of this Appendix to some illuminating conversations with my friend Mr. M. J. Farrell.

† If the differences between the observed distributions are very great it may be necessary to choose more than one class; this follows from the method outlined in the following paragraph.

The calculation of Q then requires that R be inverted, and this is not too difficult on account of the large number of zeros contained in the matrix. From (9), it follows that Q is given by PR^{-1}.

Since it is still true that

$$s_{t+1} = Ps_t = QRs_t$$

it follows on substituting from (10) that

$$s_{t+1} = Q\, s_{t+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

so that s_{t+1} is the equilibrium distribution corresponding to the corrected social mobility matrix Q.

Calculations along the above lines have been carried out on the data examined in the main text. The values found for the matrix Q are set out in Table 6, and the derived measures of the average time spent in each social class (comparable with Table 3 above) are given in Table 7.

TABLE 6

The Social Transition Matrix Adjusted for Shifts in the Occupational Structure

Class	1	2	3	4	5	6	7
1. Professional	0·457	0·116	0·036	0·023	0·010	0·000	0·000
2. Managerial	0·110	0·287	0·107	0·042	0·025	0·013	0·008
3. Higher grade non-manual . .	0·195	0·231	0·195	0·119	0·079	0·041	0·036
4. Lower grade non-manual . .	0·050	0·111	0·187	0·230	0·127	0·088	0·083
5. Skilled manual	0·128	0·187	0·351	0·419	0·483	0·393	0·364
6. Semi-skilled manual . . .	0·046	0·052	0·062	0·118	0·152	0·319	0·235
7. Unskilled	0·015	0·015	0·062	0·049	0·122	0·144	0·274

It will be seen that the calculations are in fact rather insensitive to occupational shifts of the order of magnitude dealt with here.

TABLE 7

The Average Number of Generations Spent in Each Social Class in England and in a Perfectly Mobile Society.
[Adjusted for Shifts in the Occupational Structure]

Class	*Average Number of Generations in* *England* (1)	*Mobile Society* (2)	*Column* (1) / *Column* (2) (3)
1. Professional	1·84	1·03	1·79
2. Managerial	1·40	1·05	1·33
3. Higher grade non-manual	1·24	1·10	1·13
4. Lower grade non-manual	1·30	1·15	1·13
5. Skilled manual	1·94	1·69	1·14
6. Semi-skilled manual	1·47	1·20	1·22
7. Unskilled	1·38	1·14	1·21

The largest change is in the figure of the average number of generations spent in the top class, which is here raised to 1·84 from its previous value of 1·63, and thus suggests that the amount of immobility is slightly higher than has been suggested in the main part of the text.

APPENDIX B

The Mean and Variance of the Times Spent in a Social Class

Let p_{jj}, or for short p, be the probability that a father who is now in class j will have his son also in class j.

Then $(1 - p)$ is the probability that the son will leave that class, and therefore also the expected proportion who stay in that class for one, and not more than one, generation.

Of the proportion p who stay into the second generation, the proportion p^2 will stay on to a third generation, and proportion $p(1-p)$ will leave at the end of the generation and thus will have stayed for exactly two generations.

In general, the proportion which stays exactly r generations can thus be seen to be $p^{r-1}(1-p)$.

Since everyone stays for exactly 1, or 2, or 3, ... generations, it follows that the sum of the proportions is unity; this can easily be seen, for

$$(1-p)+p(1-p)+p^2(1-p)+\ \dots\ =1$$

The average time spent can be found by weighting these proportions with the time spent, thus

$$\begin{aligned} E\{t\} &= \sum_{1}^{\infty} r\,p^{r-1}(1-p) = (1-p)+2p(1-p)+3p^2(1-p)+\ \dots \\ &= 1+p+p^2+\ \dots \\ &= 1/(1-p) \end{aligned}$$

which is the result already found in the main text on the basis of a simpler argument.

The variance of the time spent can be found as follows. First, find the mean square of the time,

$$\begin{aligned} E\{t^2\} &= \sum_{1}^{\infty} r^2\,p^{r-1}(1-p) = (1-p)+4\,p(1-p)+9\,p^2(1-p)+\ \dots \\ &= 1+3p+5p^2+7p^3+\ \dots \\ &= (1+p+p^2+p^3+\ \dots)+2p(1+2p+3p^2+\ \dots). \end{aligned}$$

Now, the terms in the second set of brackets are the derivatives of the terms in the first set of brackets; the sum of the second set is therefore the derivative of the sum of the first set. Hence,

$$E\{t^2\} = \frac{1}{1-p}+\frac{2p}{(1-p)^2}$$

To find the variance it is only necessary to subtract from this, in the usual way, the square of the mean time, and this gives

$$\begin{aligned} V\{t\} &= \frac{1}{1-p}+\frac{2p}{(1-p)^2}-\frac{1}{(1-p)^2}. \\ &= \frac{p}{(1-p)^2}. \end{aligned}$$

The standard deviation of the times is the square-root of this expression which has been given as (6) in the text.