The Concept of Statistical Freedom and its Application to Social Mobility
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Source: *Population Studies*, Vol. 9, No. 1 (Jul., 1955), pp. 82-95
Published by: Taylor & Francis, Ltd. on behalf of the Population Investigation Committee
Stable URL: https://www.jstor.org/stable/2172344

The Concept of Statistical Freedom and its Application to Social Mobility

By André Gabor

I. Foundations of the Statistical Approach

The essence of freedom consists in the absence of restraint, and an election or any other concrete instance of choosing is generally deemed to have been free if each voter had an unrestricted choice between several alternatives, of which not casting a vote may have been one.

How can we ascertain whether this has been the case or not in a given election? The most obvious method seems to be to ask each voter to what extent he felt free from restraint in making his choice, but unsurmountable difficulties would be encountered in the classification and objective evaluation of the answers.

Fortunately, there is another, hitherto neglected avenue open to us, which leads to the possibility of objective appraisal. *The extent to which a member of a group was free in choosing between a set of alternatives can be gauged by the extent to which the group as a whole availed itself of the variety of choices offered.*

This approach is objective because it is based on *post factum* data reflecting actual behaviour in a given situation, and it is statistical in so far as the analysis embraces the decisions taken by the full membership of the selected group. The result, however, may legitimately be interpreted as a quantitative expression of the freedom of the average member. If all the members of a group have made the same choice, they have not displayed any freedom in their actions, and if they have distributed themselves evenly over the several alternatives open to them, they have achieved the maximum degree of manifest freedom in the given situation.

Diversity of action is thus an essential feature of manifest freedom. To put it another way: an opportunity of which no member of a group avails himself makes no positive contribution to diversity, and therefore its effect on the measure of manifest freedom should not be positive either. If, for example, none of the diners select No. 27 from the wine list of a restaurant, it makes no difference when it subsequently turns out that bin No. 27 was in fact empty. Some people might argue that it would annoy them considerably to see an item struck off the list even though they had no intention of ordering it. So much can readily be conceded, but not the implication that an objective concept of freedom should take account of their resentment. After all, others might consider No. 27 a poor and over-priced wine, and register relief if it were removed from the wine-list! Both feelings may be fully genuine, they may have considerable influence on the momentary happiness of the people concerned, but have nothing to do with freedom in the realistic sense of the word.

But diversity alone does not necessarily indicate the absence of restraint, because variety can be enforced just as effectively as uniformity. If, for example, the actual statistical spread of the choices can be traced back to the definite orders of an authority, the actual freedom displayed is nil. Similar considerations apply if the supply of any of the alternatives is limited. Say we observe a group of twenty people as they buy their morning paper. The first day all of them buy paper A, i.e., they act uniformly and hence their actions manifest zero level of freedom. Next day the supply of paper A runs out after the first ten in the line have made their purchase, and the remainder, rather than being left without anything to read, take paper B as the only alternative. Diversity is now at the maximum possible for a case of two alternatives, but freedom is still zero as before. Even if the supply of the papers is unlimited, the same conclusion applies if it is found that the ten who chose paper A were all businessmen, say, and the other ten, who took paper B, were without exception manual workers, so that the choice of every member of the group could have been predicted from his occupation.

It follows that this approach leads to a measure of freedom which cannot sink below the zero level. *Concrete freedom can never be negative, and the utmost that coercion can do is to wipe it out altogether.*

A few examples of restraints have been quoted above, but many others are possible, and the reader will ask how far the investigator should go in his search for them before he can pronounce on the extent to which diversity represents freedom. The answer is simply that like any other statistical enquiry, the analysis of measurable freedom can settle definite questions only. It is well known that this is not a weakness but rather a great strength of the statistical method in general, because it forces the enquirer to formulate realistic and meaningful questions before expecting realistic and meaningful answers. Concrete freedom *to do something* desirable involves freedom *from being forced to do so*, and behind every demand for liberty there is the desire to escape some restraint. For these reasons *the method here discussed is so designed that it should yield a numerical measure of the freedom revealed by the behaviour of a group in a given situation, after having accounted for the influence of a set of specifically named factors.*[1]

Factors can be dichotomized in three fundamental ways: they are (i) either avoidable or unavoidable; (ii) either desirable or undesirable; and (iii) either internal or external. Let us deal with these categories in turn.

Unavoidable factors raise no moral issues; we just note that he who wants to live has to breathe, and this is the end of the matter. The proper question of freedom arises only if the restraints are avoidable. The law that in England everybody has to drive on the left *could* be altered—whether it *should* in effect be altered involves a value judgment from which the statistician *qua* statistician must refrain.

There are also instances where, though the factor itself is unavoidable, its consequences are not. Take sex, for instance, as a determinant of choice of profession. Whereas it is hardly a practical proposition to alter the sex of a girl who wants to pursue a profession at present reserved to males, it would not be impossible to throw all professions open to both sexes. Here again the existing

[1] It is preferable to speak of factors rather than restraints, for obvious reasons. Being a businessman is not a restraint in the general sense of the word, but as we have seen, it could be looked upon as a determinant of choice in the above example.

social framework must be treated as a datum by the statistician. He will record in what way, if any, sex restricts choice, and leave it to social reformers to draw the appropriate conclusions from his results.

In addition to being avoidable, a factor must also be undesirable if it is to be reckoned as a genuine restriction of freedom. The word " undesirable " might suggest that we are opening the door to subjective arbitrariness, but this is not in fact so. There exists, in every civilization, a large measure of agreement concerning the desirability or otherwise of certain restrictions ; one can even say that it is this body of tenets which constitutes the very essence of a civilization. Thus it is an accepted belief of Western thought that slavery means the imposition of evil restraints, while it is right and proper to place restrictions on the sale of dangerous poisons. The acceptance of this basis for the definition of undesirable factors saves the statistical concept of freedom from arbitrariness, but it also implies a warning : for purposes of comparison, each formulation can claim validity only within a given civilization.[1] Again this is a feature which is common to most sociological and economic concepts of statistics. For example, it is not possible to say that the standard of living enjoyed by the inhabitants of country X is higher than in country Y, except in so far as we accept the conventions of one of the two countries as the basis of the comparison.

Finally, factors can be internal, external (or both), and it must be stressed that this distinction is essentially irrelevant to any objective approach. Statistical analysis of observational data can reveal *dependence* only, i.e., association or correlation of variates, and such dependence does not involve any postulate of causality.

Most of the examples hitherto adduced represent simple cases of the " yes or no " type, seldom encountered in practice. What we generally find is neither full dependence nor complete independence, but a degree of association or correlation between factors and choices. Hence any sensible formula defining a measure of freedom must be so constructed as to be reasonably sensitive to the actual extent of dependence present.

II. Fundamental Propositions of the Analysis

We have now reached the stage where we can conveniently summarize the essence of the introductory reasoning. It amounts to the following general propositions :

1. An objective measure of freedom must yield numerical values which do not offend instinctive estimates of " greater or less " to such an extent as to be unacceptable for quantitative discussions, though some conflict with pre-quantitative ideas might be unavoidable.
2. *Post factum* data relating to instances in which every member of a group of persons made a choice from a set of alternatives can be analysed to reveal the degree of freedom which remains after having accounted for the dependence between the choices and a set of factors which are considered both avoidable and undesirable in a given civilization.

[1] " If you ask me what a free government is, I answer, that, for any practical purpose, it is what the people think so." (Edmund Burke, *Letter to the Sheriffs of Bristol*, 1777).

3. Freedom has two observable attributes, diversity and independence, and varies directly with them. If all members of a group have made the same choice, or if the diversity displayed shows perfect correlation with the factors, freedom is zero. In all other cases freedom is a positive quantity.

These propositions if accepted as conventions of the analysis are useful pointers in the search for suitable statistical formulae, but they do not lead to a unique definition. As will be shown below, these conventions can be satisfied by a variety of formulations, and the choice between them should be guided by considerations similar to those which apply to problems of measurement in general.

III. Some Basic Formulæ and Their Application to a Simple Example

The data needed for the quantitative analysis of freedom consist in the numbers of persons who selected each of the alternatives offered, and whose condition was characterized by the factors included in the investigation. These numbers are then converted into proportions, as shown in Table I, which represents an imaginary example.

TABLE I.

j \ i	1	2	$p(\,.\,j)$
1	$p(1,1)$ 0·15	$p(2,1)$ 0·30	$p(\,.\,1)$ 0·45
2	$p(1,2)$ 0·50	$p(2,2)$ 0·05	$p(\,.\,2)$ 0·55
$p(i\,.\,)$	$p(1\,.\,)$ 0·65	$p(2\,.\,)$ 0·35	$\Sigma\, p(i\,.\,) = \Sigma\, p(.\,j)$ 1·00

Let us say that the data refer to the presidential election of the Union of Students at a university. There were two candidates, denoted by $i=1$ and $i=2$, respectively. $j=1$ signifies students of arts subjects, and $j=2$ science students.[1] $p(i,j)$ is the proportion of those who selected i in condition j, thus e.g., $p(2,1)=0\cdot30$ means that 30% of the total number were arts students who voted for candidate No. 2. The table also indicates the meaning of $p(i\,.\,)$ and $p(\,.\,j)$.

How can we determine the degree of freedom latent in this information? We start from Proposition 3, which states that observable freedom consists in diversity and independence, and choose the simplest possible relationship between the three coefficients: Freedom = Diversity × Independence. Symbolically,

$$f_{i,j}=D_i\,(1-\varepsilon^2_{i,j}) \tag{1}$$

where $f_{i,j}$ = the coefficient of freedom of choice in line i, taking into consideration the factor j;

D_i = the coefficient of diversity displayed by the choices in line i;

$\varepsilon^2_{i,j}$ = the coefficient of dependence of choice in line i on the factor j; and

$1-\varepsilon^2_{i,j}$ = the coefficient of independence.

[1] Though the course pursued by each student was itself the subject of a choice at some previous stage, it is quite legitimate to treat it as a factor in this analysis, since by the time of the election it was one of the characteristics of each person.

The reason why the coefficient of independence can be written in this form is evident from Proposition 3. If the factor investigated is not found to have had any influence on the distribution of the choices, the whole of the diversity represents freedom, and if the dependence is complete, freedom is nil, however high the diversity. It follows that $1-\varepsilon^2_{i,j}$ is a suitable expression, provided $\varepsilon^2_{i,j}$ is defined so that it has 0 as its minimum value and $+1$ as its maximum. Thus it can never become negative and can legitimately be written as a quadratic, which is in line with the notation current in statistical theory.

Next we have to define D_i and $\varepsilon^2_{i,j}$. This could be done in an arbitrary way, as long as the basic conditions are satisfied, but it has been shown elsewhere that it is preferable to derive the formulæ from a mathematical model, which interprets the actual distribution of choices as the result of a maximization process.[1]

The simplest formulæ so far derived by this method are the following :

$$D_i = 1 \Big/ \sum_i \frac{p(i\,.\,)}{1-p(i\,.\,)} \tag{2}$$

$$\varepsilon^2_{i,j} = \sum_i \frac{1}{1-p(i\,.\,)} \quad \sum_j \frac{1}{p(\,.\,j)} \left[p(i,j) - p(i\,.\,)p(\,.\,j) \right]^2 \tag{3}$$

Both formulæ are quite general, in the sense that they can be extended to any number of choices or factors, be they simple or composite.[2] Expression (2) is so constructed that if all the choices are concentrated on one of the alternatives, the diversity becomes zero, and it is at a maximum if the spread of the choices is even. The value of the possible maximum varies directly with n, the number of the alternatives, the formula being $D_i(\max) = 1 - 1/n$. Expression (2) divided by this gives the *normalized* diversity :

$$D_i(\text{norm}) = \frac{D_i}{D_i(\max)} = \frac{D_i}{1 - \dfrac{1}{n}} \tag{4}$$

which indicates the extent to which the available choices have been utilized. For example, if the spread is even over two choices, the diversity is 0·5, irrespective of whether the number of alternatives offered was four or five, but since D_i (max.) is 0·75 for $n=4$, and 0·8 for $n=5$, the normalized diversity will indicate

[1] Readers not familiar with the mathematical methods of the physical and social sciences might think that erecting models which explain phenomena by alleging that they owe their existence and shape to the maximization of some potential, is just a complicated way of proceeding from an unwarranted assumption to a foregone conclusion. In fact this is not so, and mathematical models of this type have proved their worth in fields as widely different from one another as thermodynamics and economics.

In evolving the model which gave rise to the expressions discussed below, it was assumed that the actual distribution came about by maximizing the sum of two functions. The first function is a type of weighted utility sum, the second is a measure of the spread of the distribution, and the maximization of the sum brings about an optimum compromise between the two basic tendencies which could be expressed, if somewhat crudely, as the herd instinct and the individualistic drive. (For the actual derivation of the formulæ here used and for various alternatives, see Denis Gabor and André Gabor, " An Essay on the Mathematical Theory of Freedom ", *Journal of the Royal Statistical Society*, Series A, CXVII (1954), I, pp. 31–72. Cf. especially pp. 43 and ff., also p. 70. (Some of the alternative formulæ are theoretically more satisfactory but somewhat less convenient from the point of view of computation.)

[2] An example of a composite choice is newspaper *and* political vote, and of a composite factor, education *and* domicile.

that in the first case nearly 66·7%, and in the second case only 62·5% of the maximum has been attained. It is seen that the formula for D_i is in conformity with the proposition mentioned in Section I, that an opportunity of which no member of a group avails himself makes no positive contribution to diversity.

Expression (3) satisfies all the conditions laid down in the foregoing discussion. If $p(i,j) \neq p(i\,.\,)\,p(\,.\,j)$ throughout, and the choices are found to be fully predictable from the factors covered by the investigation, the coefficient of dependence is unity, and hence $f_{i,j}$ is zero, whereas if no trace of dependence is present, then $\varepsilon^2_{i,j}=0$, and $f_{i,j}=D_i$. In all other cases, the value of $\varepsilon^2_{i,j}$ is between 0 and +1. Expression (3) has also certain other satisfactory mathematical properties, foremost of which is that it can never be decreased by increasing the number of factors investigated.

For the data of Table I we obtain (corrected to the second decimal place):

$$D_i=0\cdot 42, \qquad D_i(\text{norm})=0\cdot 84, \text{ and } \varepsilon^2_{i,j}=0\cdot 36.$$

Hence

$$f_{i,j}=0\cdot 42\ (1-0\cdot 36)=0\cdot 27.$$

We can also calculate the normalized freedom coefficient:

$$f_{i,j}\,(\text{norm})=0\cdot 84\ (1-0\cdot 36)=0\cdot 54.$$

These results are interesting in themselves, though one could not say off hand whether 0·27 is a "high" or a "low" value for the coefficient of freedom under the conditions stated. But as soon as we have a second instance at our disposal, we can make a comparison, and determine without ambiguity which of the two is characterized by the greater, and which by the lesser amount of realized freedom. Say that a year later another similar election was held, which yielded the data of Table II, and let us tabulate the coefficients for both tables for comparison.

TABLE II.

j \ i	1	2	$p(\,.\,j)$
1	0·21	0·24	0·45
2	0·47	0·08	0·55
$p(i\,.\,)$	0·68	0·32	1·00

Coefficient of	Table I	Table II
Diversity ...	0·42	0·38
Dependence ...	0·36	0·17
Freedom ...	0·27	0·32

We see that although diversity decreased from the first election to the second, freedom increased, because of a substantial reduction in dependence. Though the majority of arts students still preferred Candidate 2, and the majority of the science students Candidate 1, the dependence of the vote on the course pursued by the student was less marked in the second instance.

This is as far as we can go in such simple cases. The extension of the analysis will be demonstrated presently, utilizing the data of a recent sociological enquiry.

IV. An Analysis of Social Mobility in Stockholm

Problems of social mobility provide one of the numerous fields suitable for the analysis of freedom. The files of various research organizations contain a great many relevant data, but unfortunately in most publications the information is broken up into parts and presented in the form of univariate or bivariate tables only, which exclude the possibility of studying the combined influence of the factors. However, the data of one such enquiry have been made public in the form of a tri-variate table, and will be used here to demonstrate the analysis of freedom in the choice of occupation.

In 1951 Professor Gunnar Boalt and Dr. Carl-Gunnar Jansson started a pilot investigation into the relationship between the social class of about 2,000 young men educated in Stockholm, their I.Q. obtained in the group test to which they were subjected when they appeared before the Enlistment Board, and the social class of their fathers, tracing the data back to 1936.[1] The relevant part of the information they collected is given in the table below.

The I.Q. distribution was standardized in the usual way, with $m=100$ and $\sigma=15$ for army recruits from the whole of Sweden. The 1907 Stockholm-bred young men to whom Table III refers were as a group well above the average, as indicated by the fact that their median I.Q. was approximately 113.

The classification of occupations into three social classes was done in accordance with the system used in Swedish election statistics :

Social Class I: Landowners, industrialists, manufacturers, merchants and company directors. Higher-grade employees, engineers and shop managers. Higher officials. The professions. Houseowners and other persons of independent means.

Social Class II: Farmers (whether tenants or owner-occupiers), and farmers' sons. Artisans, shopkeepers and other persons engaged in commerce or industry. Sea-captains. Persons employed in higher domestic service.

Social Class III: Farm foremen, farm hands and other agricultural labourers. Sailors and fishermen. Workers not in agriculture. Persons employed in lower domestic service.

The fathers' occupation was returned in 1936, when the sons were about ten years old, the sons' occupation was taken as found in 1949, i.e., when they were about 24 years old.

In order to reduce the number of empty cells and small frequencies, the tails of the I.Q. distributions were merged, leaving eight sub-classes, the limits of which are indicated by the division lines in Table III. The result of this operation is a $3 \times 3 \times 8 = 72$ cell table, and with the frequency in each cell divided by the total number of 1907, the material is ready for our analysis. The choice is the social class of the son, the factors are the son's I.Q. and his father's social class, which will be designated by i, j and k, respectively.

Let it be said in advance that we are not going to argue here to what extent intelligence and father's occupation should or should not influence the occupation pursued by the son. What we propose to do is to provide objective measurements which can serve as a basis for discussion, and to show that the coefficients introduced in Section III are acceptable for this purpose.

[1] *Social Mobility in Stockholm*. Paper read before the Liège Congress of the International Sociological Association, August, 1953. I am greatly indebted to the authors for permission to use their data.

TABLE III

Son's I.Q.	Number of Sons	Father in Social Class I			Father in Social Class II			Father in Social Class III		
		Son in Social Class			Son in Social Class			Son in Social Class		
		I	II	III	I	II	III	I	II	III
> 153	1	1								
147—153	6	2			2	1			1	
140—146	25	8	1		9	2		2	3	
133—139	96	31	4		25	21		8	7	
126—132	215	42	17		37	54	5	6	41	13
119—125	302	28	29	1	28	92	12	2	87	23
112—118	388	11	43	1	7	96	18	4	135	73
105—111	349	2	12	1	1	72	28		110	123
98—104	291	1	10	1	2	43	25		97	112
91— 97	143		4			11	17		28	83
84— 90	60		2			7	4		13	34
77— 83	21						2		7	12
70— 76	6								2	4
63— 69	2					1				1
52— 62	2								1	1
Total	1907	126	122	4	111	400	111	22	532	479

The diversity in the sons' choice of social class is

$$D_i = 0{\cdot}542\,; \qquad \text{and} \qquad D_i\,(\text{norm}) = 0{\cdot}813.$$

This is not very different from the diversity of the distribution of the fathers:

$$D_k = 0{\cdot}550\,; \qquad \text{and} \qquad D_k\,(\text{norm}) = 0{\cdot}825,$$

because the discrepancy between the two distributions is mainly due to compensating changes in Classes 2 and 3. The mobility displayed is in fact much greater than the extent of upgrading generally observed in steadily progressive communities, and calls for an explanation. Professor Boalt has pointed out that between 1936 and 1949 large numbers of young people moved from country districts into Stockholm, and that these "immigrants" tended to enter the less desirable occupations vacated by the families whose sons were receiving education in Stockholm schools in 1936. The approximate percentages given below illustrate the position.

	Social Class 1	2	3
"Fathers" in 1936 ...	13·2%	32·6%	54·2%
"Sons" in 1949	13·6%	55·3%	31·1%
"Immigrants" in 1949 ...	4·3%	27·4%	68·3%
All Stockholm	7·9%	39·7%	52·4%

There is clear indication here that the group of the " sons " enjoyed what may be termed a privileged position relative to the " immigrants ". It is regrettable that further data are not available in respect of the latter, because comparisons between the two groups would be of far greater interest than the analysis of the Stockholm educated group alone.

The dependence of social class on I.Q. is revealed by the coefficient

$$\varepsilon^2_{i,j} = 0\cdot146,$$

which means that about 14·6% of the diversity displayed can be accounted for by reference to I.Q. ratings, and hence the diversity qualified by the influence of this factor is

$$f_{i,j} = D_i(1 - \varepsilon^2_{i,j}) = 0\cdot542\,(1 - 0\cdot146) = 0\cdot463.$$

The formula for ε^2 is not symmetrical, in the sense that generally $\varepsilon^2_{i,j}$ is *not* equal to $\varepsilon^2_{j,i}$. The former measures the extent to which we can reason from I.Q. to social class, the latter reverses the relationship between choice and factor. In this instance

$$\varepsilon^2_{j,i} = 0\cdot051.$$

It is general experience that the social selection process is more effective in eliminating persons of low intelligence from the highly graded occupations than in ensuring a place at or near the top for all who have a high I.Q. The fact that $\varepsilon^2_{i,j}$ is larger than $\varepsilon^2_{j,i}$ does not contradict this principle, but it is certainly interesting to note that as far as this specific group is concerned, we can infer the I.Q. rating of a man from the knowledge of his social class with less certainty than the other way round.

The influence of the father's social class on the son's is less than that of the son's I.Q., the value of the coefficient of dependence being

$$\varepsilon^2_{i,k} = 0\cdot082.$$

The dependence of I.Q. on the father's social class is even lower. We find

$$\varepsilon^2_{j,k} = 0\cdot022.$$

We shall examine presently to what extent the absolute and relative magnitudes of these coefficients are attributable to the grouping of the data. As it is, the evidence points towards the conclusion that the fathers were rather more successful in keeping their sons in their own social class than in providing them with what may be termed the I.Q. appropriate to their class.[1]

It is interesting to note also that the dependence of the father's social class on the son's I.Q. is greater than the dependence of I.Q. on father's social class :

$$\varepsilon^2_{k,j} = 0\cdot107,$$

which is larger than

$$\varepsilon^2_{j,k} = 0\cdot022.$$

Inspection of the details of the computation suggests that the difference $\varepsilon^2_{k,j} - \varepsilon^2_{j,k} = 0\cdot085$ is mainly attributable to the $k = 3$ sub-class.

[1] The heavy overlap between the I.Q. ranges of social classes as displayed by Table III is an oft observed phenomenon. This feature seems to have been partly responsible for the low values of those coefficients of dependence in which j is included.

We now turn to the combined effect of the factors, which is not simply the sum of the two coefficients. The proper answer is provided by the logical extension of Expression (3) to cover the case of the composite factor *jk*. The formula is

$$\varepsilon^2_{i,jk} = \sum_i \frac{1}{1 - p(i\,.)} \sum_j \sum_k \frac{1}{p(\,.jk)} \left[p(i,jk) - p(i\,.)\; p(\,.jk) \right]^2 \quad . \quad (5)$$

This expression can further be extended to cover any higher number of factors.

Applying Expression (5) to the data, we obtain

$$\varepsilon^2_{i,jk} = 0{\cdot}208.$$

This is smaller than $\varepsilon^2_{i,j} + \varepsilon^2_{i,k} = 0{\cdot}228$. The difference of 0·020 may be termed the *interaction* between the fathers' occupation and the sons' I.Q. Since the combined dependence is in this case smaller than the sum of the two separate coefficients, the interaction is negative, which is in conformity with the dependence we found between the I.Q. of the son and the social class of the father. $\varepsilon^2_{i,jk} - \varepsilon^2_{i,j} = 0{\cdot}208 - 0{\cdot}146 = 0{\cdot}062$ is the *net* addition to the dependence by the fathers' social class.

The freedom which remains after having accounted for the dependence of the choice of occupation on both factors combined is

$$f_{i,jk} = D_i\,(1 - \varepsilon^2_{i,jk}) = 0{\cdot}542\,(1 - 0{\cdot}208) = 0{\cdot}429,$$

and $$f_{i,jk}(\text{norm}) = D_i(\text{norm})\,(1 - \varepsilon^2_{i,jk}) = 0{\cdot}813\,(1 - 0{\cdot}208) = 0{\cdot}644.$$

It would be futile to ask whether the final figure represents "true" freedom or not. What statistical freedom claims to be is neither more nor less than that part of the diversity which cannot be explained with the data at our disposal. As mentioned in Section I,[1] concrete freedom in any given sphere must be interpreted as freedom from definite restraining factors which are considered undesirable in a given civilization. The statistical analysis is indifferent to ethical judgments, but the interpretation of the results need not be so. Prevailing opinion appears to consider the influence which the father's social class exerts on the son's choice of occupation on the whole as undesirable, and the influence of the son's intelligence on the same choice as desirable. (It was for this reason that $f_{i,j}$ was given the neutral description of qualified diversity on page 85.) It is of course generally recognised that on the one hand I.Q. is a somewhat unreliable indicator of the elusive entity we call intelligence, and on the other hand that neither I.Q., nor intelligence proper (whatever that may be) can alone decide whether a person is truly suitable for a given occupation or not, and also that intellectual suitability for a certain job does not necessarily tally with the wishes of the individual concerned. Any further discussion of these and related issues would be outside the scope of this paper.

It is conceivable that if we knew more about each of the 1907 young men than just his I.Q. and the social class of the father, we could explain a greater portion of the diversity. In fact our formulæ are so constructed that drawing further factors into the analysis can never increase the freedom, and unless the new factor is perfectly correlated with one already considered, the effect will be a reduction in the measure of freedom. It would seem then that if we push the analysis of each case to its potential limits, in the end freedom will always

[1] Cf. p. 82 above.

disappear altogether. Though this may be perfectly true, there is no need to fear that we shall ever know enough of the external and internal conditions of individual persons to be able to predict *all* their choices with full certainty. In the case of single lines, however, such as votes in elections, it would not be surprising to find that with the knowledge of a few conspicuous characteristics, such as occupation, education, etc., we can occasionally account for a major part if not for the whole of the diversity.

V. Effects of Grouping on the Values of the Coefficients

The values of our coefficients are not entirely independent of the grouping of the variates. This is not a peculiarity of the formulæ introduced in this analysis ; it applies quite generally to *all* statistical measures used in connection with grouped distributions. The influence of the grouping can be studied by varying the number of the divisions, which in the case of the data in hand is practicable with respect to I.Q., i.e. j only.

The finest reasonable sub-divisions of the I.Q. distribution are those used in Section IV, and the alternatives consist in merging the sub-classes by pairs and in forming two groups of four each.

These groupings give new values to all those coefficients which cover j. The results are tabulated below, together with the appropriate figures of Section IV for comparison.

Coefficient	Number of j sub-classes 2	4	8
$\varepsilon^2_{i,j}$	0·076	0·133	0·146
$\varepsilon^2_{j,i}$	0·221	0·110	0·051
$\varepsilon^2_{k,j}$	0·075	0·096	0·107
$\varepsilon^2_{j,k}$	0·123	0·046	0·022
$\varepsilon^2_{i,jk}$ ...	0·145	0·186	0·208
$f_{i,j}$	0·501	0·470	0·463
$f_{i,jk}$	0·463	0·441	0·429
$f_{i,jk}$(norm) ...	0·695	0·662	0·644

The above table enables us to study the effects of grouping both ways, in so far as j, i.e., the I.Q. of the sons, figures in a formal way as a "choice" in $\varepsilon^2_{j,i}$ and $\varepsilon^2_{j,k}$, and as a factor in the other three coefficients of dependence. The former increase with finer sub-division of j, and the latter decrease, which is as it should be. In order to conform to fundamental requirements which apply to all such measures, the formula of the coefficient of dependence is so constructed that it should never decrease with finer sub-division of the factor or factors, and although finer sub-division of the choices may increase it at first, it will always decrease if the sub-division is pushed beyond a certain point. The figures well illustrate these properties, except that the values of $\varepsilon^2_{j,i}$ and $\varepsilon^2_{j,k}$ do not display the range of increase which occurs with some data.

It should not surprise us to see that the relative magnitudes of $\varepsilon^2_{i,j}$ and $\varepsilon^2_{j,i}$, and also of $\varepsilon^2_{k,j}$ and $\varepsilon^2_{j,k}$ change order when only two sub-classes of

i are distinguished. We know that coarse grouping often covers a multitude of sins, though in this case the reversal of the order is not so much a defect as rather a logical consequence of the procedure. Here is a further warning that if comparisons are made, like must be compared with like.

VI. The Weighting of Choices

Hitherto the analysis was entirely neutral to the order of the social classes. Though we denoted the top class by 1, the centre class by 2 and the lowest class by 3, we have not thereby introduced any ranking, and the results would remain unchanged in the face of any re-numbering. We know, however, that the social significance of the three classes is not by any means equal, and it seems apposite to give expression to this fact by *weighting* the numbers in each cell before calculating the proportions.

Annual money income is a possible basis for such a weighting system. Perhaps it is not the best, but there is certainly much to recommend it. Though money may not be the most important thing in life, it is a fairly good indicator of the social value associated with most occupations. It is also an objective magnitude, i.e., one which can be determined fairly easily with an accuracy which is sufficient for the purpose, and since only relative incomes matter, weights based on them may be used even in international comparisons. But whatever the merits or demerits of income weights may be, the main purpose of introducing them here is to illustrate the use of the method.

Professor Gunnar Boalt has obligingly provided the present author with some unpublished data relating to the income distribution of parents in 1936, from which the following approximate values were derived:

	Social Class			
	1	2	3	
Average income p.a.	24,000	6,000	2,000	Sw.Cr.
Proportionate weight	12	3	1	

Since the purpose of the weighting is to give to each of the sons' choices the appropriate relative significance, the weights were applied according to i, though the above averages really relate to k. This procedure involved the probably not unjustifiable assumption that no appreciable change in the *relative* incomes of the three social classes had taken place between 1936 and 1949.

The method was as follows: the frequency in each cell was multiplied by 12, 3 or 1, according to the i sub-class, and then expressed as a fraction of the weighted total. The weighted and unweighted coefficients for the $3 \times 3 \times 4$ table are shown here in juxtaposition:

Coefficient	Weighted	Unweighted
D_i	0·563	0·542
D_i(norm)	0·844	0·813
$\varepsilon^2_{i,j}$	0·280	0·133
$\varepsilon^2_{i,k}$	0·211	0·082
$\varepsilon^2_{j,k}$	0·067	0·046
$\varepsilon^2_{k,j}$	0·089	0·096
$\varepsilon^2_{i,jk}$	0·371	0·186

Coefficient	Weighted	Unweighted
$f_{i,j}$	0·405	0·470
$f_{i,jk}$	0·354	0·441
$f_{i,jk}$(norm)	0·531	0·662

$\varepsilon^2_{k,j}$ is the only coefficient of dependence which the weighting failed to increase. This is not surprising, since, as mentioned above,[1] the comparatively high value of the unweighted $\varepsilon^2_{k,j}$ was largely attributable to the $k=3$ sub-class, the relative importance of which was reduced by the weighting from about 54% to about 34% of the total. It is also worth noting that the increase in the other coefficients is far from proportionate, and as the relative change in $f_{i,j}$ was greater than in $f_{i,jk}$, the difference between them, i.e. the net reduction in freedom attributable to the influence of the fathers' social class, which was approximately 6·2% in the absence of weighting, increased to 12·6% when measured with the weighted coefficients. The general inference is that weighting is a useful expedient, provided the weights have been appropriately selected, and that just as in the case of index numbers, its effect is not predictable in advance.

VII. Freedom and Randomness

The relatively coarse grouping of occupations into three broad classes enabled us to by-pass an important issue, namely the superficial similarity between freedom and mere randomness. Young men choosing between occupations frequently show indifference between two or more of them, and wherever this is the case, that portion of the diversity which owes its existence to the toss of a mental coin, as it were, should be eliminated at the outset. Since in this example the great majority of those sets of occupations within which individuals could be supposed to have been indifferent were not split by the boundaries, it seemed justifiable to apply the diversity as found. But problems of this type cannot always be solved by making a virtue out of a necessity. When it comes to problems of freedom in the disposal of income, in the choice of political party, etc., only the persons themselves are fully qualified to say if and in what way the alternatives should be merged. Therefore if there is no reliable social scale to guide the grouping, additional information is necessary, which may be obtained in two different ways. The first is to question the individuals, i.e., to present to them a full list of the choices, and ask them to bracket together those items between which they are indifferent, or, alternatively, ask them which if any of the other items they would select if their favourite choice were not available. The second method applies only if the choice is of the recurrent type, and observation can be extended over a longer period, during which each of the alternatives can be temporarily withdrawn, one at a time, so that the effects can be observed in turn. (There are pitfalls in any of these methods, too well known to need enumeration here, but they are not worse than those which beset the collection of similar data for other purposes.) Once we have got a list of secondary choices, we can eliminate that part of the diversity which is attributable to randomness.[2]

[1] Cf. p. 90. Though the remarks there refer to the $3 \times 3 \times 8$ table, they also apply to the $3 \times 3 \times 4$ case.
[2] Cf. D. Gabor & A. Gabor, *loc. cit.*, p. 52.

VIII. Epilogue

In this age in which we are nearer than ever before to the attainment of freedom for all from the want of basic necessities, and in which freedom of one kind or another figures prominently in the slogans and battle cries of most political creeds, the need for an objective concept of freedom could hardly be denied. It has been shown in the foregoing that freedom has a measurable aspect, and that appropriate methods can be designed to analyse its nature and extent. It has also been shown that the methods here proposed are both versatile and capable of further development, and that the stage is set for further theoretical work and practical research to proceed *pari passu*.

The actual lines of further progress must depend on the nature of the problems thrown up by studies in the social sciences, just as the evaluation of the results of the analysis must be left to students of social ethics. It has been well said by Sir Arthur Bowley that "*the statistician's contribution to a sociological problem is only one of objective measurement, and this is frequently among the less important of the data; it is as necessary, however, to its solution as accurate measurements are for the construction of a building*".[1] The theory of quantitative freedom as here presented represents a step in the direction of objective measurement, and the author hopes that the tools here introduced will be found useful by those for whose purposes they were designed.

[1] A. L. Bowley, *Elements of Statistics*, London, 1926, p. 13.