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SOCIAL MOBILITY AND FERTILITY *

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In several recent studies the effects of mobility or status inconsistency on a dependent variable have been quantified by means of an an additive model in which sets of constants have been fitted to two principles of classification. In examining a particular application of this model, the following paper begins by suggesting the possibility that the underlying hypothesis may be more adequately represented by a symmetrical model which fits one and the same set of constants to both principles of classification.

The second purpose of the paper is to show that, whether or not the symmetrical model is deemed to be the more appropriate, *the basic hypothesis can be adequately tested only by the formulation of likely alternatives and the employment of tests which are specific to those alternatives.*

Thirdly, a consideration of two alternatives to the basic model—one of which is simply a linear transformation of the other—implicitly demonstrates that some of the problems (of multicollinearity or identification) which are associated with quantitative studies of difference variables such as inconsistency or mobility are analogous to the pseudo-problems generated by the concept of rotation in factor analysis.

The generalization of the methods employed to more than two principles of classification and to more than one dependent variable is obvious.

Preamble **

In their work on *The American Occupational Structure* Blau and Duncan (1967) devote a number of pages to a discussion of what they call "the mobility hypothesis," particularly to the form [1] in which it was advanced by R. A. Fisher in *The Genetical Theory of Natural Selection.* Various formulations of the hypothesis are cited. It is claimed that the hypothesis is refuted if the data exemplify a particular pattern, which they term "the additive hypothesis." In this paper data which have previously been held to satisfy the additive hypothesis are re-examined to see whether in fact they satisfy that hypothesis, either in its original form or in a modified form.

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** This preamble grew out of comments and criticisms on the following sections of the paper which were made by Mrs. Jean Floud and Professor O. D. Duncan. As a reward for my attack on his hypothesis, Professor Duncan has, with his usual generosity, supplied me with data on which further studies of fertility and mobility may be carried out. Although we appear to disagree on several points, he and I are in entire agreement on the need to replicate findings such as those reported here. The additive hypothesis, in an approximate form, has already stood up to several replications and is to that extent on a surer footing than the mobility effect which I claim to detect.

[1] In considering their argument, it is important to note that the mobility which Blau and Duncan subject to empirical test is mobility of the present generation. They make only passing reference to the Galton-Fisher hypothesis of the inheritance of (voluntary or involuntary) infertility, which is a mechanism whereby the mobility of an antecedent generation might affect the fertility of the following generation. This restriction is apparent in their argument that if differential fertility were completely explained by social mobility then there would be no differential fertility by class among persons who do not change their class.

The analysis is carried out entirely within the terms laid down by the preceding work, and the form of the analysis is a repetition and extension of that work. These limitations have been observed quite deliberately in order to ensure that the two sides of the argument come adequately to grips with one another.

It should, however, be said here that the Blau and Duncan argument might perhaps be side-stepped altogether by a refusal to acknowledge that the truth of the additivity hypothesis disposes of the social mobility hypothesis in another sense of the term. It might be said that, given that fertility decreases with increasing social class among the nonmobile, the truth of the additivity hypothesis implies that those who rise out of a particular class are less fertile and those who fall out of the same class are more fertile. Sociologists might feel that the establishment of a simple additive law would have a miraculous quality which would cry out for intensive investigation of the mechanisms which bring it about. Indeed, the fact that the additive hypothesis is even approximately true leads one to wonder how it is that the values and life-styles of former and newly-encountered social aggregates lawfully modify conception-decisions, when movement between those aggregates appears so various in its abruptness, its finality, its extent, its salience and its temporal relations to the child-bearing period.

To investigate the social mobility hypothesis, as they have defined it, Blau and Duncan carry out four analyses, three of which yield significant departures from additivity. They say that they are not satisfied to observe that significant deviations $\bar{Y}_{ij} - \hat{Y}_{ij}$ occur and that it must be shown that these deviations are in some systematic way related to the notion of mobility (1967:377). In this paper the challenge is taken up in a twofold sense. (1) We begin by reanalyzing data which do not, by Blau and Duncan's criterion, show significant departures from additivity, and we show that by a more appropriate criterion such departures are in fact present. (2) We show that the departures are systematically related to the notion of mobility, indeed that they instantiate that form of the mobility hypothesis with which additivity is incompatible. The nub of the argument is that the criterion of departure from additivity is blunt-edged: it lumps together likely and unlikely departures in such a way that the former are swamped by the latter. The sociologist and the nonsociologist (e.g., Fisher) alike, faced with a table which relates mobility to fertility (such as Table 1 below), would begin by distinguishing the mobile from the nonmobile and the upwardly-mobile from the downwardly-mobile. He would ask whether mobility as such is related to fertility, whether direction of mobility affects fertility, and whether fertility varies with extent of movement. To quote Westoff's discussion (1956), "In its most simple outline, there is a three-point continuum: upward mobility, immobility or stability, and downward mobility. In addition to direction, there is the question of intensity or degree of movement." These questions are posed below as explicit alternatives to the additive hypothesis. They are not factitious consequences of a desperate search for significance. On the contrary, they arise naturally and have arisen in advance of any empirical examination of the facts.

Introduction

Empirical studies of the relations between fertility and social mobility have led demographers to induce that the average fertility of those who move up or down the socioeconomic scale appears to be intermediate between that of the class from which they came and that of the class into which they have moved (Maxwell, 1953: 101n; see also Blau and Duncan, 1967: Chapter 11). Precision may be given to this observation by replacing the word "intermediate" by the word "halfway." The first purpose of this paper is to design a model which adequately expresses the proposition that the fertility of socially mobile couples is halfway between the fertility of the class from which they remove and the fertility of the class into which they move. The second purpose of the paper is to suggest alternatives to the hypothesis whose empirical plausibility may be assessed as a means of testing the basic hypothesis, which we shall call the Halfway hypothesis.

In the following two sections we begin by considering a model which Duncan employed

Table 1. Mean Number of Live Births per Couple, by Present Social Class and Class of Origin of Husband.

Husband's Origin	Present Social Class I	II	III	IV	All
I	1.74	1.79	1.96	2.00	1.81
II	2.05	2.14	2.51	2.97	2.38
III	1.87	2.01	2.67	3.69	2.81
IV	2.40	3.20	3.22	3.68	3.44
All	1.88	2.17	2.73	3.56	2.77

Table 2. Numbers of Couples from Which the Means in Table 1 Were Derived.

Husband's Origin	Present Social Class I	II	III	IV	All
I	65	43	23	11	142
II	38	197	150	68	453
III	37	154	431	244	866
IV	5	45	162	220	432
All	145	439	766	543	1893

to test for the existence of a mobility effect on fertility. This model is regarded as not entirely appropriate to the test of the Halfway hypothesis though it is closely related to the model which is found to be more appropriate, and the two do not differ significantly in their degree of fit to the data which are subjected to analysis.

Duncan refers to his model, which is called here Model 1, as an exemplification of the additivity hypothesis. The refined version which constitutes our second model (Model 2) exemplifies what we call the Halfway hypothesis. These names may cause some confusion because both models are in fact additive. Our purpose in presenting these two, slightly differing, models is to challenge theorists to define the hypothesized consequences of their theories so precisely that the empirical worker can decide *a priori* which model better represents a certain hypothesis.

The particular (additivity) model which is investigated below is of wider interest than our concentration on a single study of fertility might suggest, since it is relevant, for example, to any study whose aim is to decide whether a person with two discrepant sources of status stands halfway between persons who are crystallized at his higher level and other persons who are crystallized at his lower level (e.g. Hodge and Treiman, 1966). Generalization of the model to three or more stratification axes is a simple matter.

The Data

The relations between social mobility and fertility were investigated by Berent (1952) in a paper which Duncan (1966) has described as "the only worthwhile discussion of its subject." Berent's data were derived from a sample representing the population of England and Wales taken in 1949. He reported the mean fertilities[2] of 1893 married couples, classified by the occupational class of the husband's father and the occupational class of the husband himself (Tables 1 and 2). He also presented an analysis of variance of these data. Duncan repeated the analysis of variance—or "multiple classification analysis," as it is sometimes known, perhaps in allusion to the title of Yates's (1934) original paper on analyses of variance with unequal numbers in the cells—and he reports the constants of the model which is implicit in Berent's analysis. On the basis of this analysis Duncan concludes that there is no need to postulate an effect of social mobility on fertility. He finds that the fertliity of a couple may be regarded as a combination of the fertility of their class of origin (class of husband's father) and the fertility of their destination class (husband's own class). There is no significant deviation of the observed mean fertilities from those estimated on the basis of this hypothesis. Duncan's analysis is employed in the following section to give a lead in to the test of the Halfway hypothesis.

The First Model

In his analysis Duncan assumes that a class can have two fertilities, one when it is an origin class and the other when it is a destination class. His model, which is the first to be considered here, may be written:

[2] Number of live births to couples married for more than twenty years, neither member having been previously married.

$$\hat{Y}_{ij} = \mu + a_i + b_j,$$

where $\hat{Y}_{ij}$ is the estimated mean fertility of couples in origin class i and destination class j; $\mu = 2.77$ represents the general mean; a_i represents the fertility of the ith class of origin; and b_j represents the fertility of the jth destination class. Table 3 shows that the sum of squares of the deviations from the model—i.e., the sum of squares of $(Y_{ij} - \hat{Y}_{ij})$, where Y_{ij} represents an observed cell mean in Table 1—is 66.72.[3]

This model has been employed in a very similar analysis of comparable American data in Blau and Duncan (1967: chapter 11). They argued that it is plausible to assume that there are separate effects for the two statuses because differential fertility is observed when couples are classified either by husband's first job or by current occupation (1967: 374). The argument from differential fertility for both origin and destination implies that both should be taken into account in arriving at an estimate of the effect of class on the fertility of a married couple. However, it does not imply that, because origin and destination each has its effect, the magnitude of a class effect should differ according to whether it is an origin or a destination class. A supplementary proposition is required to justify the postulation of two separate effects for one and the same class. One such supplement might run as follows: class i considered as an origin will differ from class i considered as a destination because the former antedates the latter by a period of time which may be quite lengthy.

[3] It should be noted that Berent gives the between cells sum of squares as 762.8, whereas Table 3 shows that it should be 729.60+66.72=796.32. Berent's error may imply a corresponding overestimation of the within cells sum of squares (if that was obtained by subtraction), but an error of this magnitude in the within term is negligible. Alternatively, there may be a slight error in the data as reported.

Table 3. Mean Squares for First Model: $\hat{Y}_{ij} = \mu + a_i + b_j$

Source	df	SS	MS
Constants	6	729.60	121.60
Residual	9	66.72	7.41
Within Cells	1877	9311.50	4.96
Total	1892	10107.82	

The Second Model

It might, however, be thought that the class effects will be roughly constant over time, and a second model, incorporating this assumption, may be constructed:

$$\hat{Y}_{ij} = \mu + c_i + c_j$$

Model 2 thus fits a constant effect for a class whether that class is an origin or a destination class. Naturally, the explained sum of squares for the second model is less than that for the first, but there is a compensating increase in the residual degrees of freedom. In the first model eight constants are fitted, six of them being independent. In the second model four constants are fitted, three of them being independent.

We may assess the importance of the difference between the two models by testing the discrepancy between the residuals; this discrepancy yields a sum of squares of $89.68 - 66.72 = 22.96$ with 3 degrees of freedom. This is not significant, but it is sufficiently large to make us hesitant about accepting the hypothesis of no difference. The marginal constants for the two models are given in Table 4. It should be observed that this second model is a subset of the first in that the sum of squares explicable by the two models acting together is identical with the sum of squares attributable to the first model alone.

On the whole the second model seems to be a better interpretation of the proposition

Table 4. Constants of Two Models:
(1) $\hat{Y}_{ij} = \mu + a_i + b_j$
(2) $\hat{Y}_{ij} = \mu + c_i + c_j$

Social Class	First Model		Second Model
	a	b	c
I	-.58	-.60	-.59
II	-.20	-.50	-.35
III	-.01	-.07	-.03
IV	.42	.66	.55

$\mu = 2.77$

that the fertility of a couple is an additive combination or averaging of the fertility of their class of origin and that of their class of destination. This may be inferred from a comparison of the estimates of fertility which emerge from the two models. The second model, the one which ascribes a single effect to each class, yields a symmetric matrix of estimates, that is, it implies that the fertility of couples moving up from i to j is the same as the fertility of couples moving down from j to i. The first model, the one which assigns two constants to each class, does not lead to symmetric estimates. The proposition in its verbal form seems to imply symmetry in any model which purports to embody it.

Furthermore, the second model, unlike the first, yields an estimate of fertility for couples moving from i to j which is the unweighted mean of the estimate for couples remaining in i and the estimate for couples remaining in j. For example, the estimated value for stationary[4] couples in class one is 1.598, that for stationary couples in class two is 2.076, and the estimate for those who move, in either direction, between these two classes, is 1.837.

The Effect of Mobility on Fertility

If we accept that Model 2 is an adequate formulation of the Halfway hypothesis, it remains for us to test the fit of the hypothesis to the data. In fitting a three-dimensional model to fifteen-dimensional[5] data, we are left with a large number of respects in which deviations from the model might occur. It is desirable, therefore, to examine the hypothesis to see whether any deductions may be drawn from it which may be tested in some specified subset of the residual twelve dimensions. The subset which we shall choose is one which has the property of isolating possible effects of mobility on fertility, after allowing for the class effects incorporated in the model. (The word "effect" in the term "mobility effect on fertility" is used here in its technical sense. The finding of a mobility effect would not imply that mobility is the *cause* of differences in fertility.)

A single dimension along which deviations from the Model might occur is that contrasting the mean of all stationary couples with the mean of all mobile couples. Let us, therefore, fit a constant m which has a positive value for the diagonal cells of Table 1 and a negative value for the remaining cells. Adding this constant to the equation for Model 2 augments the explained sum of squares by only 2.84, which is certainly not significant. A similar constant added to the equation of Model 1 augments the sum of squares by 1.67.[6]

Two further contrasts may be drawn, one between the upwardly mobile and the rest, and the other between the downwardly mobile and the rest. Since comparison among the three means (those of the upwardly mobile, the downwardly mobile and the immobile) can be made in a space of only two dimensions, it is convenient to combine these last two contrasts into a single contrast between the upwardly mobile and the downwardly mobile, with the deviation of the nonmobile from the mean of the mobile taken care of by the constant m. Let us use d for the constant which expresses the effect of direction of movement; d has a positive value for the downwardly mobile and a negative value for the upwardly mobile. Clearly, it stands for a classical sociological effect. Hawthorn and Busfield (1968: 193) report Berent's analysis in support of the conclusion that fertility is negatively associated with upward mobility and positively associated with downward mobility. But Hawthorn (1970: 109) later accepted Duncan's interpretation of Berent's data in which no mobility effect, equivalent to our constant d, is postulated.

We have now constructed a simple alternative to the second model by adding two

[4] That is, stationary so far as the data allow us to determine. A couple may have moved in and out of a class several times between the two points in time for which their class is recorded. And it must be remembered that the status of a couple is defined entirely by reference to the husband's origins and occupation, omitting any reference to the wife.

[5] The 16 means in the body of Table 1 may be regarded as points generating a space of 15 dimensions.

[6] The absence of a contrast between the mobile and the nonmobile does not necessarily instantiate the Halfway hypothesis because the mean of the mobile is based on more downward than upward movers.

constants to the original set. The alternative takes the forms:

for the upwardly mobile $(i > j)$
$$\hat{Y}_{ij} = \mu + c_i + c_j - m - d$$
for the nonmobile
$$\hat{Y}_{ii} = \mu + 2c_i + m$$
for the downwardly mobile $(i < j)$
$$\hat{Y}_{ij} = \mu + c_i + c_j - m + d$$

The sum of squares accounted for by the alternative model is 736.26, which exceeds the sum of squares attributable to the second model by 29.62. Tested against the within mean square[7] of 4.96, this yields a variance ratio of $F_{2,1877} = 2.99$, which lies almost exactly at the 5% point. If we test d against all the remaining effects (including m), we obtain a sum of squares of $736.26 - 709.48 = 26.78$ which, with one degree of freedom, is significant at the 2½% level. Clearly, the sum of squares remaining to m is of no account.

The introductory discussion of the two models, although it tended to suggest that the second is a better embodiment of the Halfway hypothesis, did suggest a rationale for the first, namely that the fertility effect of a class may vary over time and so should be estimated separately for the class considered (1) as an origin and (2) as a destination.[8] We may, therefore, ask whether the mobility effect (which has been demonstrated when the hypothesis is schematized in the second model) is abolished if we reincorporate into the analysis the effects which are taken into account by the first model. Let us, therefore, construct an alternative to the first model. Employing the letters m and d as before, we write:

for the upwardly mobile $(i > j)$
$$\hat{Y}_{ij} = \mu + a_i + b_j - m - d$$
for the nonmobile
$$\hat{Y}_{ij} = \mu + a_i + b_i + m$$
for the downwardly mobile $(i < j)$
$$Y_{ij} = \mu + a_i + b_j - m + d$$

The sum of squares for the first model is 729.60 (Table 3). When m is added, this becomes 731.28. The sum of squares for the alternative, including both m and d, is 753.89. Once again, there is significant evidence of a difference between the fertility of the upwardly mobile and that of the downwardly mobile ($F_{1,1877} = 4.56$; $p < .05$).

Whichever model we take as representative of the effects of class on fertility, we arrive at the conclusion that there is a mobility effect over and above the class effect. The mobility effect takes the form of higher fertility for the downwardly mobile and lower fertility for the upwardly mobile, with the nonmobile scarcely deviating from the weighted mean of the two. In the alternative to the second model, the extent of this mobility effect on fertility is $\pm d = \pm 0.1665$ of a child, a discrepancy of one third of a child, on average, between the upwardly mobile and the downwardly mobile. The remaining values of the constants in the alternative to the second model are shown in Table 5. In the alternative to the first model $\pm d = \pm 0.0512$ of a child, a discrepancy of one tenth of a child between the upwardly and the downwardly mobile. It appears, therefore, that the first model comes closer to accounting for the mobility effect than does the second, but neither succeeds completely.

We have now disproved the Halfway hypothesis by showing that, whichever model we choose to represent it, a mobility effect which is inconsistent with the hypothesis can

Table 5. Values of the Constants in Model Which is Alternative to the Second Model.

μ	c_1	c_2	c_3	c_4	m	d
2.77	-.59	-.35	-.03	.54	-.03	.17

[7] It is convenient to employ a constant error term in all tests of significance, and the within mean square is a good enough approximation when the value of R^2 (the ratio of the between cells to the total sum of squares in Table 3) is as low as 0.08. If the reader prefers an error term which errs on the conservative side, he may substitute the total mean square, which is 5.34. It is not possible to take account of the effect of the sampling design on variances.

[8] Other mechanisms which would imply the first model may easily be imagined. These take the form of supposing that a class is more salient to conception in one of its manifestations than it is in the other, for example as origin rather than destination, as higher rather than lower, or as some combination of the terms of these two dichotomies.

be detected. It should be made clear, however, that what we have proved is the existence of a mobility effect as a deviation from the Halfway hypothesis. In order to establish the existence of a mobility effect *simpliciter,* we fit the model:

for the upwardly mobile $(i > j)$
$$\hat{Y}_{ij} = \mu - m - d$$
for the nonmobile
$$\hat{Y}_{ij} = \mu + m$$
for the downwardly mobile $(i < j)$
$$\hat{Y}_{ij} = \mu - m + d$$
and we find that
$$m = -.0293$$
$$d = .2194$$

and the explained sum of squares is 49.78 with two degrees of freedom, which establishes the existence of a mobility effect and shows that the original interpretation of the data is justified.

The Halfway hypothesis and the mobility effect hypothesis differ in the extent of their falsifiability, the former being disproved by a wider array of circumstances than the latter. If we reduce the precision of the Halfway hypothesis by reverting to some such proposition as that the fertility of mobile couples tends to lie somewhere between the fertility of their class of origin and the fertility of their destination class, then, it might be argued, the undoubted explanatory power of the hypothesis may be salvaged. The difficulty with this formulation is that the explanatory power of the hypothesis can be assessed only if it is expressed in a precise form, and it is not clear on what basis a model of this formulation (which is much vaguer than Duncan's hypothesis) could be constructed. A better statement of our conclusion is that the Halfway hypothesis comes close to being true, but its applicability is modified by the existence of a mobility effect.

The Third Model

In the preceding sections we have referred to the constants a, b and c as "class effects." This is not a felicitous term. Consider a_1 and a_4 in Table 4. The former is directly derived from the fertility of couples who have stayed in class one or moved downward from class one into other classes. The latter is directly derived from the fertility of couples who have stayed in class four or risen from it into other classes. No upwardly mobile couples contribute to a_1, and no downwardly mobile couples contribute to a_4. Similar observations may be made on the b coefficients. It is only because Tables 1 and 2 are roughly symmetrical that each a_i is roughly similar to the comparable b_i, and c_i is close to both.

So far we have treated Table 1 as a row x column analysis of variance table. Let us now rid ourselves of this paradigm, turn the table through 45°, and treat the diagonal as the axis of major interest, that is, the set of four cells containing the nonmobile couples. If we are to find class effects in the table, they should be estimated by these cells. We must, of course, assume either that there is little movement out of the classes and back into them again, or that the effects of such movements are balanced out.

The mental activity of turning our attention from the rows and columns to the principal diagonal of the fertility table may be paralleled by a similar change in our model, a change which factor analysts would call a rotation to a new set of axes. It has been observed that the four constants of Model 2 lie in a three-dimensional space, one of them being redundant. We shall, therefore, in the course of performing the rotation, reduce the number of constants from four to three. The transformation matrix for carrying out the proposed rotation is furnished by the set of orthogonal polynomials for a set of four points:

	linear	quadratic	cubic
c_1	-3	1	-1
c_2	-1	-1	3
c_3	1	-1	-3
c_4	3	1	1

As an example of the employment of this matrix let us look at the equation for estimating the contents of cell two/three in the second model. This may be written,
$$\hat{Y}_{23} = \mu + 0c_1 + 1c_2 + 1c_3 + 0c_4$$
Multiplying the coefficients in this equation by each of the columns of the transformation matrix in turn yields the equation,
$$\hat{Y}_{23} = \mu + 0 \text{ linear} - 2 \text{ quadratic} + 0 \text{ cubic}$$
It happens that all the coefficients of the new model are even numbers and so the

model may be more succinctly expressed by halving each coefficient.

It is of interest to write out the equations for the diagonal cells explicitly (the terms "linear," "quadratic," and "cubic" are replaced by x_1, x_2 and x_3, respectively):

$$\begin{aligned}\hat{Y}_{11} &= \mu - 3x_1 + x_2 - x_3\\ \hat{Y}_{22} &= \mu - x_1 - x_2 + 3x_3\\ \hat{Y}_{33} &= \mu + x_1 - x_2 - 3x_3\\ \hat{Y}_{44} &= \mu + 3x_1 + x_2 + x_3\end{aligned}$$

The equation of the estimate $\hat{Y}_{ij}$ for any off-diagonal cell, $i \neq j$, is the mean of the equations for the appropriate diagonal cells $\hat{Y}_{ii}$ and $\hat{Y}_{jj}$.

It must be made quite clear that this, third, model is merely the second model in a new guise. The degree and nature of the fit to the data are the same and so are the estimates which the model yields.

The differences between the estimated and the observed values for the diagonal cells have a sum of squares of 11.06, which is not significant. The estimates derived from the model may, therefore, be taken as estimates of class effects on fertility.

Apart from its conceptual simplicity, and the reduction in the number of constants, an advantage of this model is that it points to ways of achieving further simplification. The sum of squares attributable to the linear component is 671.75. When the quadratic element is added, this rises to 704.40. The addition of the cubic element adds only a further 2.24, and this element may be ignored.

The main advantage of introducing this model which is no more than an algebraic variant of its predecessor is that it comminutes our natural set towards a two-dimensional table, which leads us to treat it as a row by column table in the analysis of variance. Changing the standpoint from which we view the table is of psychological, though not logical, assistance in that it suggests an alternative model which differs from the alternative to Model 2. Although it is perhaps placing too much weight on a single table of means to subject it to another set of tests, it is nevertheless instructive to spell out the new alternative hypothesis and show how it may be tested.

Our former alternative hypothesis considered only two compendious contrasts, the one between mobile and nonmobile, and the other between upwardly mobile and downwardly mobile. The new alternative hypothesis takes into account extent, as well as existence and direction, of mobility. Having established the principal diagonal of Table 1 as the axis of primary sociological interest and as the base axis for generating the constants of the Halfway hypothesis, we now take the dimension at right-angles to that diagonal as specifying the first dimension of deviation from the hypothesis to be explored. By assigning to the cells the weights shown in Table 6 we, in effect, impose an axis along which the cells are distributed according to the direction *and extent* of movement of their members. Nonmovers are at the point of balance; those who move one class down are one step to the right of the nonmovers; those who move one class up are one step to the left of the nonmovers, and so on as far as the most mobile couples who move three class steps in one direction or the other. This axis is a linear polynomial which distinguishes number of classes moved but does not distinguish between, say, those who move from class one to class two and those who move from two to three or three to four. Subsequent axes defined by higher-order polynomials also have these two properties. The second, quadratic, polynomial is given in Table 7. The reader may readily construct the remaining tables by consulting a table of orthogonal polynomials such as that in the *Biometrika Tables for Statisticians*.[9]

[9] In practice it is convenient to generate orthogonal polynomials by a computing routine. It should be observed that the second set of polynomials is orthogonal to the first. Since the first

Table 6. Constants Defining a Possible Linear Effect of Direction and Extent of Mobility on Fertility.

Husband's Origin	Present Social Class I	II	III	IV
I	0	1	2	3
II	-1	0	1	2
III	-2	-1	0	1
IV	-3	-2	-1	0

Table 7. Constants Defining a Possible Quadratic Effect of Direction and Extent of Mobility on Fertility.

Husband's Origin	Present Social Class I	II	III	IV
I	-4	-3	0	5
II	-3	-4	-3	0
III	0	-3	-4	-3
IV	5	0	-3	-4

Taking first the linear deviations representerd by Table 6, we find that these contribute a sum of squares of 13.85 over and above that accounted for by the linear and quadratic elements of the third model. (We have already decided that the cubic term of the model may safely be ignored because its sum of squares is only 2.24.) The linear component of the alternative model has 1 degree of freedom and is, therefore, not significant. The quadratic component contributed by the constants in Table 7 adds only a negligible 1.57 to the explained sum of squares. This is not surprising, since the main contrast provided by the term is between the immobile or the one-step movers on the one hand, and the long distance movers on the other. We have already seen that the fertility of the mobile does not differ appreciably from that of the nonmobile.

set has three members and the second set has six members, the space spanned by the axes of the alternative model has nine dimensions. In the analysis no two polynomials remain quite orthogonal when fitted because the number of couples differs from cell to cell in no systematic manner; nevertheless the analysis remains nine-dimensional. The degree of overlap between the two sets of polynomials may be gauged from the fact that the sum of squares for the first set is 706.64, that for the second set is 72.54, and the sum of squares jointly explained by the two sets is 755.57.

Table 8. Constants Defining a Possible Cubic Effect of Direction and Extent of Mobility on Fertility.

Husband's Origin	Present Social Class I	II	III	IV
I	0	1	1	-1
II	-1	0	1	1
III	-1	-1	0	1
IV	1	-1	-1	0

The third, cubic, component, whose constants appear in Table 8, is significant, contributing a sum of squares of 23.23 over and above the combined effects of the earlier terms in the analysis. The regression coefficient for this term is positive. An examination of Table 8 shows what this means: the model ascribes high fertility to five out of the six sets of downwardly mobile couples and it ascribes low fertility to five out of the six sets of upwardly mobile couples. The two exceptions are the two smallest cells in the table, containing 16 couples in all (Table 2). This, cubic, term is identical with the d term of the alternative to the second model, apart from the weights assigned to the two extreme cells. The two analyses have, therefore, arrived at practically identical conclusions. If the linear term of the second alternative model had proved to be significant, this would have indicated that degree, as well as direction, of social movement is related to fertility.

A further three orthogonal polynomials may be fitted but, taken together, these add no more than 10.54 to the explained sum of squares, from which we may infer that none of them considered individually can attain significance.

It may be desirable to indicate in more detail the nature of the conclusion we have demonstrated from our exploration of an alternative to the third model. We have established the significance of only one term, namely the cubic, but the first linear term, although not significant, is not negligible. In any statement of conclusions it is unwise to ignore lower-order terms with moderately high sums of squares because the sequential nature of the testing implies that each higher-order term is to be considered as a deviation from, or modification of, the earlier terms. The conclusions of the analysis may, therefore, be represented as in Figure 1, which incorporates a straight line with a small positive slope to indicate the existence of a possible linear component on which the downwardly mobile have a higher projection than the upwardly mobile (the regression coefficient of the linear component is positive). The cubic component represents a degree of wobble about this line, but the

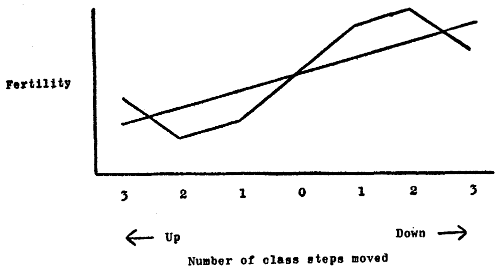

FIG. 1. Schematic representation of the linear and cubic components of the effect of social mobility on fertility.

overall effect is again to contrast the two directions of mobility.

The Two Forms of Alternative Hypothesis

We have examined two possible alternatives to the basic model (Model 2 or its equivalent Model 3)[10]: one in terms of effects m and d, and the other in terms of polynomials. An examination of the constants of the two alternatives has led us to suppose that there is considerable overlap between them, and this is borne out by the calculation of the additional sums of squares which they explain, acting jointly and severally. The addition to the basic model of the linear and cubic components of the alternative adds 38.25 to the explained sum of squares. The addition of m and d adds 29.26. The addition of all four terms adds 43.55, which is little more than the contribution of the polynomials acting alone.

Summary of the Analyses

Our analysis of the third model and its alternative has reiterated the conclusions which we reached in explaining the second model and its somewhat cruder atlernative. Both analyses result in the rejection of the Halfway hypothesis and in the acceptance of an alternative which involves an effect of direction of social mobility on fertility, the downwardly mobile being more fertile than the upwardly mobile, with the non-movers in the middle.

The third analysis has added a couple of refinements to the conclusion of the second. The first refinement consists in the observation that the class effects specified by the Halfway hypothesis may be represented in only two dimensions, although four classes have been defined. The two dimensions are a linear ranking of the classes in their expected order and a quadratic element which may reflect no more than the particular choice of cut-off points between neighboring classes.

The second refinement is the suggestion that the d effect of the second alternative has a flick in its two tails, namely the deviation of the extremely mobile from the pattern of those who move in the same direction but to a less extreme degree. These tails are based on very few couples and their anomalous values are best ignored. Indeed, the anomaly is in fact confined to only one of the tails, namely that consisting of the highly downwardly mobile. If this effect were to recur in an analysis of further fertility tables, we should probably want to examine individual cases before offering an explanation.

As an indication of the nature of the effects which have been demonstrated, Table 9 contains the deviations of the estimated from the observed values when the estimates are derived from the Halfway hypothesis (ignoring the cubic term in the specification of the hypothesis) and Table 10

Table 9. Observed Minus Estimated Mean Fertilities of Couples, Where the Estimates Are Derived From the Halfway Hypothesis.

Husband's Origin	Present Social Class I	II	III	IV
I	.08	-.05	-.24	-.76
II	.21	.13	.13	.04
III	-.33	-.37	-.07	.39
IV	-.36	.27	-.08	-.17

[10] It should be noted that the polynomial deviations for Model 3 could well have been tested as deviations from Model 2 since (so long as we retain all three polynomials which specify Model 3) the two models are completely equivalent, yielding the same prediction for each cell of the data and hence explaining the same proportion of the overall sum of squares. Many of the difficulties which research workers find themselves in when handling difference concepts such as mobility or inconsistency would dissolve if they would learn to ask not "are these two sets of equations identical?" but "are the *spaces* mapped by these two sets of equations identical?"

Table 10. Observed Minus Estimated Mean Fertilities of Couples, Where the Estimates Are Derived From an Alternative to the Halfway Hypothesis.

Husband's Origin	Present Social Class I	II	III	IV
I	.06	-.25	-.33	-.09
II	.40	.13	-.06	-.03
III	-.26	-.17	-.06	.21
IV	-1.02	.35	.13	-.16

contains similar values for the alternative hypothesis which incorporates the linear and quadratic elements of the Halfway hypothesis plus the linear and cubic elements of the deviations. It will be observed that, when the values in Table 9 are subtracted from those in Table 1, the result is a symmetrical table of estimates such that the fertility of couples moving from i to j is the same as the fertility of couples moving from j to i.

The residual sum of squares for the Halfway hypothesis, so specified, is 91.92 with 13 degrees of freedom; that for the alternative (including the linear as well as the cubic component) is 53.51 with 11 degrees of freedom. If this reduction seems unimportant, it should be borne in mind that the mean deviation of the downwardly mobile from the Halfway hypothesis is 0.1897, and that of the upwardly mobile is 0.1431, giving a difference between the two of 0.3314, or one third of a child. (This difference is almost but not quite identical with twice the value of d in the alternative to the second model.) In quantitative terms this means that the 539 downwardly mobile couples have 102 children more than the Halfway hypothesis implies, and the 441 upwardly mobile couples have 63 children fewer than the hypothesis implies, a difference of 165 in a sample of approximately 5250 children.

APPENDIX: A NOTE ON CRITICISMS AND QUERIES

A number of enquiries and criticisms have been made concerning the methods and conclusions of this paper, and some of these are of sufficiently general import to merit brief discussion. I shall deal with the criticisms first and then explain the technical procedures which have aroused interest.

One criticism which has been levelled against the paper is that the Halfway hypothesis, as schematized in Model 2, is too rigid and has less sociological justification than Model 1. My reply is that this may be true, but the proposition as commonly stated (a) does not specifically allow that a class may have more than one effect, and (b) appears to say that the fertility of those moving from class i to class j is literally halfway between the fertility of those in i and the fertility of those in j. To the best of my knowledge, this paper is the first to point out that two models are possible and, in consequence, it is the first to recognise the need for, and provide, possible sociological justifications of the first model. Sociologists are now free to name their preferred model in the light of the preceding explorations of the properties of the two schematizations. Both models must be supplemented by a mobility effect.

A second criticism has been that examination of the residuals of either model (for example the deviations from Model 2 reported in Table 9) does not reveal uniformly positive values for the downwardly mobile and negative values for the upwardly mobile. This is obviously true: it is unlikely that eye-assessment of residuals would suggest the existence of the d effect. It is not clear, however, what we are supposed to infer from the criticism. Do we throw overboard the concept of the variance of a mean when some means have the temerity to pass over to the wrong side of the magic number zero? This cannot be taken seriously, particularly when the deviations in question are based on fairly small numbers. Alternatively, the criticism might imply that a more complex effect than a simple contrast between the upward and downward movers is at work; but it was at finding just such an effect that the second (polynomial) alternative model was aimed, without any real success. It should be evident that any relatively small effect is going to be consistent with the presence of deviations with "wrong" signs.

A third criticism is that some of the deviations of the observed cell means from the means given by the (basic plus) alternative models are quite large and some are greater than the deviations of the same means from the means given by the basic model (Model

1 or Model 2). My reply is that the requirement that the fit should improve in any and every cell is unreasonable and its nonfulfilment is not sufficient to invalidate a model. The greatest deviations (whether enlarged or reduced by the addition of the alternative model) occur in the cells with the smallest numbers of couples and, therefore, have the largest standard errors and contribute only moderately to the residual sum of squares.

Following upon this criticism is the suggestion that one should examine individual cells to see if any sense can be made of the deviations from a proposed model. While not entirely rejecting this recommendation, I would point out that, if results of any generalizability are to be achieved, it is desirable to start by postulating effects which (a) have a basis in sociological theory and (b) span as many of the data as possible, so that their fit is to the generality of the available evidence rather than to certain aspects only. (Of course, the possibility of satisfying (b) may be circumscribed by a degree of specificity in the theory mentioned in (a).)

Some queries have arisen over the nature of the polynomials employed as an alternative to Models 2 and 3. The co·nputing procedure for applying values such as those in Tables 6, 7 and 8 is described below. Here it is sufficient to point out that a variable such as fertility may be linearly related to class (if we are looking at the basic model) or to extent of mobility (if we are looking at the alternative model), but higher-order polynomials are required to supplement the linear polynomial because our categorization of class or extent of mobility has not produced equal-interval groupings. Linearity is always relative to some scale.

An objection has been raised to the fitting of the d effect in the first alternative model. Why, it has been said, should we not split it into two separate effects: (d_1) the difference between the downwardly mobile and the nonmobile, and (d_2) the difference between the upwardly mobile and the nonmobile? It will, of course, be observed that only two degrees of freedom are available for tests of differences among the three mobility categories. This means that the sum of squares explained by m and d will be the same as the sum of squares explained by m, d_1 and d_2. (The latter analysis is necessarily singular, but singularity is a feature of several of the models in this paper and is easily handled by the methods described below.) The objection overlooks the fact that the sum of squares attributable to m has been shown to be very small indeed. From this we infer that the nonmobile lie at the weighted mean of the downwardly mobile and the upwardly mobile, and so all the remaining sum of squares (whether computed as the sum of squares due to d or the sum of squares due to d_1 and d_2) must be due to the fact that the downward movers lie on one side of the mean and the upward movers on the other. With allowance for the moderate excess of downward over upward move·s, we may say that downward and upward movement have equal and opposite effects on fertility.

Lastly, it has been asked how in fact it is possible to ensure that the constant for row i of a square table of means is the same as the constant for column i, and it is also asked how the singularity of several of the analyses is handled. To help us appreciate the answers to these questions, let us first look at the computing procedure which was employed in the repetition of Duncan's analysis.

The following design matrix is con-

cell i, j	n_{ij}	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
1,1	65	1	0	0	0	1	0	0	0
1,2	43	1	0	0	0	0	1	0	0
1,3	23	1	0	0	0	0	0	1	0
1,4	11	1	0	0	0	0	0	0	1
2,1	38	0	1	0	0	1	0	0	0
2,2	197	0	1	0	0	0	1	0	0
2,3	150	0	1	0	0	0	0	1	0
2,4	68	0	1	0	0	0	0	0	1
3,1	37	0	0	1	0	1	0	0	0
3,2	154	0	0	1	0	0	1	0	0
3,3	431	0	0	1	0	0	0	1	0
3,4	244	0	0	1	0	0	0	0	1
4,1	5	0	0	0	1	1	0	0	0
4,2	45	0	0	0	1	0	1	0	0
4,3	162	0	0	0	1	0	0	1	0
4,4	220	0	0	0	1	0	0	0	1

structed for the table (Table 2), n_{ij} is the number of couples in the cell in row i and column j (Table 2). An 8 x 8 sums or squares and sums of products matrix **W** is computed from the eight columns of the design matrix, employing the n_{ij} as weights. This matrix is singular in that its last two latent roots are necessarily zero. Let us write λ_i for the i^{th} latent root of $\mathbf{W}$. Then we may write the well-known equality,

$$\mathbf{W} = \Sigma\, \lambda_i\, \mathbf{w}_i\, \mathbf{w}'_i$$

here the $\mathbf{w}_i$ are column latent vectors with unit sums of squares, and the $\mathbf{w}_i\, \mathbf{w}'_i$ are matrix products known as unit hierarchies. It is perhaps not so well-known that,

$$\mathbf{W}^k = \Sigma\, \lambda^k_i\, \mathbf{w}_i\, \mathbf{w}'_i$$

where k is any power, and in particular that [11]

$$\mathbf{W}^{-1} = \Sigma\, \lambda_i^{-1}\, \mathbf{w}_i\, \mathbf{w}'_i$$

We use this last equation to construct a pseudo-inverse for $\mathbf{W}$ by calculating its latent roots λ_i and their associated latent vectors $\mathbf{w}_i$ and, omitting both zero roots, computing,

$$\mathbf{W}_*^{-1} = \Sigma\, \lambda^{-1}\, \mathbf{w}_i\, \mathbf{w}'_i$$

where i runs from 1 to 6.[12]

We now calculate the vector $\mathbf{p}$, each of whose eight elements is a sum of products for a variable of the design matrix and the observed cell mean fertilities weighted by the n_{ij}.

[11] A simple arithmetic example of these equations is given in the hardbacked edition of K. Hope (1968:193). *Methods of multivariate analysis.* University of London Press.

[12] The asterisk indicates a changed form of $\mathbf{W}$.

Then the calculation,

$$\mathbf{W}_*^{-1}\, \mathbf{p}$$

gives us the constants of Model 1, which Duncan obtained by the conventional method.

The computing procedure described avoids all problems of multicollinearity and singularity and has a number of incidental advantages. The writer has adopted some variant of it in all his computer programs which call for a regression analysis. (Programmers are, however, warned that design matrices frequently throw up sums of squares and sums of products matrices with unhappy properties, and tests must be made and compensating procedures introduced).

Now, in order to obtain a single set of constants c_i for the two principles of classification, we simply average or sum corresponding a_i and b_i in the above table. The first quarter of the new design matrix then has the following appearance:

cell i, j	n_{ij}	c_1	c_2	c_3	c_4
1,1	65	2	0	0	0
1,2	43	1	1	0	0
1,3	23	1	0	1	0
1,4	11	1	0	0	1

The sums of squares and sums of products matrix for this design matrix has one zero root. The generalization to several principles of classification is obvious.

Finally, we may look at the design matrix for Model 3 and its alternative (some of the polynomials for the alternative being given in Tables 6 – 8).

cell i, j	n_{ij}	x_1	x_2	x_3	linear	quadratic	cubic	quartic	quintic	sextic
1,1	65	−3	1	−1	0	−4	0	6	0	−20
1,2	43	−2	0	1	1	−3	−1	1	5	15
1,3	23	−1	0	−2	2	0	−1	−7	−4	−6
1,4	11	0	1	0	3	5	1	3	1	1
2,1	38	−2	0	1	−1	−3	1	1	−5	15
2,2	197	−1	−1	3	0	−4	0	6	0	−20
2,3	190	0	−1	0	1	−3	−1	1	5	15
2,4	68	1	0	2	2	0	−1	−7	−4	−6
3,1	37	−1	0	−2	−2	0	1	−7	4	−6
3,2	154	0	−1	0	−1	−3	1	1	−5	15
3,3	431	1	−1	−3	0	−4	0	6	0	−20
3,4	244	2	0	−1	1	−3	−1	1	5	15
4,1	5	0	1	0	−3	5	−1	3	−1	1
4,2	45	1	0	2	−2	0	1	−7	4	−6
4,3	162	2	0	−1	−1	−3	1	1	−5	15
4,4	220	3	1	1	0	−4	0	6	0	−20

It should be evident that, when the n_{ij} are employed as weights, the covariances of the above columns will not be zero in general, and it may therefore be asked why we speak of "orthogonal" polynomials. The answer is quite simply that the columns may be (and, in the case of the colunms x_1 to x_3, explicitly were) produced by applying a matrix of orthogonal polynomials to a given design matrix. The uneasiness which may be felt about the nonorthogonality of the columns has its roots in the fear that something may be lost by failure to employ uncorrelated axes. This fear is, however, groundless because (a) k correlated axes may, and frequently do, lie in k dimensions; in other words correlation is not a sufficient condition of singularity, and indeed the presence of substantial correlation is consistent with nonsingularity, and (b) the manner of arriving at the columns, by orthogonal transformation with a complete set of polynomials, ensures that the space of the original design matrix is preserved in its entirety.

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