# SOCIAL MOBILITY AND SOCIAL STRUCTURE: SOME INSIGHTS FROM THE LINEAR MODEL *

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*The matrix equation representing relationships between social structure and social mobility is reviewed, and some of these relationships which are usefully formulated, described, or clarified in terms of the elements of the model are cited. A distinction is drawn between problems in which social mobility processes are the dependent variable and those in which social structure is the dependent variable; some variants of the model are suggested for dealing with each. Limitations of the model entailed by its assumption of closed populations are noted. A model is suggested which represents the sociodemographic process transforming social structure, and describes both social mobility and differential population growth.*

We have a much more profound understanding of the relationship between social mobility and social structure, and a much more sophisticated approach to studying this relationship, than we did only a few years ago. One of the main reasons—indeed perhaps *the* main reason—for this progress has been the attempt to represent the relationship between social structure and social mobility in the form of a mathematical model, and the analysis of some of the implications of such representations. This section contains a brief review of the general form of the most important of the models suggested, the matrix equation wherein a set of mobility rates or transition probabilities is represented as the transformation carrying a vector representation of the social structure at some initial time into a new vector representation of the social structure at a subsequent time. In the next section some of the relationships between social mobility and social structure which may be deduced from such matrix equation models are reviewed. Some of the problems concerning the relationship between social mobility and social structure which are suggested by, or better formulated in terms of, such models are also noted. In the third section some additional directions and possible variants of these are suggested, and in the fourth section a linear model of social mobility, population growth, and changing social structure is presented.

The matrix equation representation of the relationship between social structure and social mobility was suggested independently by S. J. Prais,[1] and by I. Blumen, M. Kogan, and P. J. McCarthy.[2] In this model, the initial structure of a population by social class, occupational, industrial, or residential categories at some initial time, say time 0, is represented by a row vector, $a_0 = (a_1, a_2, \ldots, a_i, \ldots a_n)$ of proportions, $a_i$, of the population in the respective classes or categories $(i = 1, 2, \ldots, n)$, which are, in turn, exhaustive and mutually exclusive $(\sum_i a_i = 1)$. The set of mobility rates or transition probabilities, $p_{ij}$, of an individual in the population who is initially (at time $t=0$, or at the beginning of the first interval in question) in the *i*-th category being in the j-th category at the end of the first interval (at time $t=1$) for an n x n matrix,

$$M = [p_{ij}] \text{ where } (i, j = 1, 2, \ldots, n) \text{ and } \sum_i p_{ij} = 1.$$

Finally the structure of the population by

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[1] S. J. Prais, "Measuring Social Mobility." *Journal of the Royal Statistical Society,* Series A, 118 (1955), pp. 56–66; and S. J. Prais, "The Formal Theory of Social Mobility," *Population Studies,* 9 (1955), pp. 72–81.

[2] Isadore Blumen, Marvin Kogan, and Philip J. McCarthy, *The Industrial Mobility of Labor as a Probability Process* (Cornell Studies in Industrial and Labor Relations, No. 6), Ithaca, N. Y.: Cornell University Press, 1955.

social class, occupational, industrial, or residential categories at the end of interval in question (at time $t=1$) is represented by a second vector, $\alpha_1 = (a_1', a_2', \ldots, a_i', \ldots a_n')$ of proportions, $a_i'$, of the population in the respective categories. The representation of the relationship between social structure and social mobility [3] is, then, the matrix equation.

$$\alpha_0 M = \alpha_1 \qquad (1)$$

Many of the concepts and measures used in the study of social mobility, e.g. "elite mobility," "occupational inheritance," "migration streams" or "index of association", are readily expressed in terms of elements $a_i$, $a_i'$, or $p_{ij}$ of the matrix equation (1). This model has been widely cited as the "Markov chain model" of social mobility: the analyses of Prais are concerned with the parameters of patterns of intergenerational social mobility represented as Markov chain transition probability matrices, while the Blumen, Kogan and McCarthy study seeks to ascertain the extent to which patterns of intragenerational industrial mobility conform to or deviate from Markov chains. But the matrix equation representation of the relationship between social mobility and social structure is an interesting and suggestive model regardless of whether the pattern of mobility rates represented by $M=[p_{ij}]$ is *or is not* assumed (or found) to conform in time to that of a Markov chain. Regardless of the assumptions or absence of assumptions concerning M, the matrix equation (1) has a representation of an initial social structure, $\alpha_0$, transformed, as it were, into the new or altered social structure, $\alpha_1$, by the operation of a social mobility process, $M=[p_{ij}]$; and as such the equation (1) is at once a concise and, it will be contended in the next section, an instructive representation of the relationship between social mobility and social structure.

## RELATIONSHIPS BETWEEN SOCIAL MOBILITY AND SOCIAL STRUCTURE

In this section we list, as concisely as possible and with little or no discussion, some of the relationships between social mobility and social structure suggested, clarified or deduced from the kind of model reviewed in the previous section. In addition certain problems concerning relationships between social mobility and social structure can be seen to receive more rigorous formulation when cast in terms of the matrix equation elements.

(1) Social mobility may be associated with either stability or with change in social structure. Thus, where

$$\alpha_0 M = \alpha_1, \text{ and } M \neq I$$

we may have either $\alpha_1 = \alpha_0$ or $\alpha_1 \neq \alpha_0$. (I denotes the identity matrix.)

(2) Different patterns of social mobility may be associated with the same change in social structure. Thus, although $\alpha_0 M = \alpha_1$ we may also have $\alpha_0 A = \alpha_1$, $\alpha_0 B = \alpha_1$, and so forth, where A,B,M, etc., are different.[4]

(3) The volume of total mobility or of any combination of specific types or streams of mobility is a function of both the initial social structure (the elements $a_i$ of $\alpha_0$) and the pattern of mobility (the transition probabilities, $p_{ij}$.) [5]

(4) It is possible and useful to characterize patterns of social mobility, say $M_1$, $M_2$, ..., $M_\nu$, in terms of parameters which are independent of the respective initial social structures in connection with which they are observed. Two ways in which this may be achieved are first, the use of a "standard" initial social structure, say $\alpha_s$, and characterization of volume of total or of combinations of specific types of mobility under conditions $\alpha_s M_1$, $\alpha_s M_2$, ..., $\alpha_s M_\nu$, respectively,[6] and second, the application of the theory of Markov chains to derive asymptotic or "equilibrium state" or "stationary state" parameters of the mobility patterns, $M_1$, $M_2$, ... $M_\nu$.[7]

[3] J. Matras, "Comparison of Intergenerational Occupational Mobility Patterns: An Application of the Formal Theory of Social Mobility," *Population Studies,* 14 (November, 1960), pp. 163–169.

[4] J. Matras, "Differential Fertility, Intergenerational Occupational Mobility, and Change in Occupational Distribution: Some Elementary Interrelationships," *Population Studies,* 15 (November, 1961), pp. 187–197.

[5] Matras, "Comparison . . . ," *op. cit.*

[6] O. D. Duncan, "Methodological Issues in the Analysis of Social Mobility," in Neil J. Smelser and Seymour M. Lipset, eds. *Social Structure and Social Mobility in Economic Development,* Chicago: Aldine Publishing Co., 1966, pp. 51–97.

[7] Matras, "Comparison . . . ," *op. cit.*

(5) Analyses of the kinds noted in the previous paragraph may be invoked to show that patterns of social mobility have properties which, on the one hand, are independent of the initial social structures in association with which they are observed, and, on the other hand, have very important consequences in terms of both the changes in social structure, and the nature and volume of social mobility which they generate.[8]

(6) Two distinct types of problem of social mobility in relation to social structure[9] are brought into focus by the representations of equation (1):

(a) The nature of the mobility process as affected, influenced, or bounded by the changing social structure. In this type, the focus is on M or upon its elements, $p_{ij}$, as the dependent variable, with the initial and subsequent social structures, $a_0$ and $a_1$, as the independent variable.

(b) The implications of the mobility process, or of the operation of the mobility process upon a specific initial social structure. In this type, the focus is upon $a_1$ or its elements, $a'_j$, as dependent variables, with initial social structure and social mobility process, $a_0$ and M, as the independent variables.

(7) With respect to the former type of problem, (a), it follows from paragraph (2) above that specific changes in the social structure do not themselves completely specify the mobility process, i.e., for given $a_0$, $a_1$, there is no unique M such that $a_0 M = a_1$. On the other hand, specified changes in an initial social structure do *bound* the social mobility process in at least some respects.[10]

(8) The latter type of problem, (b), the analysis of social-structural implications and consequences of mobility processes, has received little systematic attention. However operation of a particular mobility process upon a given initial social structure does completely specify a resulting subsequent social structure. Indeed, any given series of successive mobility processes occurring over some set of successive time intervals likewise uniquely specifies some ultimate social structure;[11] i.e., given $a_0$ and M, it is possible to deduce $a_1$; and given $a_0$ and a series, $M_1$, $M_2$, . . ., $M_k$, of social mobility processes operating successively in the following k time intervals, it is possible, in general, to deduce

$$a_k = a_0 M_1 M_2 \cdots M_k = a_0 \prod_{\nu=1}^{k} M_\nu \qquad (2)$$

(9) Performance of computations as in equation (2) above, and comparison of deduced and actual social structures at time $t=k$, indicate that it is useful to partition populations or societies, and to study distinct mobility processes operating upon the separate parts of the population.[12]

(10) Change in social structure entails *both* differential growth of the various social categories *and* social mobility, two processes which may be viewed and represented separately and investigated either separately or jointly, in the form

$$a_0 D_F M = a_1 \qquad (3)$$

where $D_F$ is a diagonal matrix representing differential growth of the various social categories and M is, as before, the matrix of transition probabilities or mobility rates over the different social categories.[13]

(11) Complete social immobility can occur only under very extreme and unlikely conditions. From equation (3) above it can be seen that immobility ($M=I$) must imply either that there is *neither* differential growth of the different social categories *nor* change in the social structure ($a_1$ necessarily $= a_0$), or that there *is* differential growth of the different social categories, and that change in the social structure occurs *exclusively* as a consequence of such differential growth. The latter implies, in turn, either the eventual extinction and disappearance of certain social categories or classes, or else sharp fluctuation in patterns of differential growth over successive time periods.[14]

(12) When the pattern of social mobility, $M_1$ is viewed as the dependent variable, equation (2) shows that given changes in the social structure, $a_0$ and $a_1$, in combina-

[8] *Ibid.*

[9] Matras, "Differential Fertility . . . ," *op. cit.*

[10] *Ibid.*

[11] *Ibid.*

[12] Blumen, Kogan, and McCarthy, *op. cit.* See also L. A. Goodman, "Statistical Methods for the Mover-Stayer Model," *Journal of the American Statistical Association,* 56 (December, 1961), pp. 841–868.

[13] Matras, "Differential Fertility . . ." *op. cit.*

[14] *Ibid.*

tion with given differential growth, $D_F$, entail *minimum* amounts and types or directions of social mobility, where the minimum proportion mobile equals $\frac{1}{2}\sum_i |a_i - a'_i|$; this is often denoted "structural" or "structurally induced" mobility.[15] The residual (difference between "total" and "structural" social mobility) may be derived; it is sometimes denoted "exchange" mobility. There is some evidence that the extent of "exchange" mobility varies among different countries less than does the extent of "structural" mobility.[16]

### FURTHER DIRECTIONS AND POSSIBLE VARIANTS OF THE LINEAR MODEL

Considering separately the two types of problem of social mobility in relation to social structure, in which the social mobility process and social structure respectively are the dependent variables, we may note some general directions for further analyses based upon the matrix equation representation of this relationship. Looking first at the problem of deducing or estimating the elements of the social mobility process, $p_{ij}$, given the nature of the initial and the subsequent (transformed) social structures, $a_0$ and $a_1$ respectively, it seems clear that formulation or imposition of additional conditions or principles governing the social mobility process must operate to impose additional restrictions or bounds on the magnitudes of elements or parameters of the unknown transition matrix, M, i.e., to "close in" on the matrix. A very interesting attempt to solve M by reference to other information, knowledge, or hypotheses concerning mobility is the representation by G. Carlsson of the intergenerational mobility process as a product-matrix involving (i) the differential access to educational opportunities of sons of fathers of the several social classes, denoted by a transition matrix, say E; and (ii) the differential access of persons of different levels of educational achievement to the different classes of occupational status, denoted by a second transition matrix, say F. The social mobility matrix then is

$$M = EF \qquad (4)$$

where the matrices E and F are known.[17]

Another procedure for estimation of M, an intergenerational mobility matrix, by invoking information or principles in addition to the known $a_0$ and $a_1$, is derived by H. C. White as the consequence of (a) some known specific proportions or amounts of intergenerational "occupational inheritance," and (b) random allocation of all sons to all the rest of the occupational positions, i.e. those not "inherited" intergenerationally.[18]

The approach can be applied much more generally, with various types of information, hypotheses or empirical generalizations applied. These, in connection with knowledge of the $a_0$ and $a_1$, would specify or bound the entries of the social mobility matrices, M. For example, the assumption (i) that any individual is more likely to remain in his father's (or in his own initial) social or occupational status category than to move to some other category, and (ii) that the probability of being in or entering a given social or occupational status is higher for sons of fathers in that status category (or for persons who themselves were initially in that category) than for those originating in any other status category imposes the conditions

$$p_{ii} \geqq p_{ij} \text{ for all i, } j \neq i$$
$$\text{and } p_{ii} \geqq p_{ki} \text{ for all i, } k \neq i \qquad (5)$$

in addition to the usual transition matrix equation restrictions,

$$\sum_i p_{ij} = 1 \text{ for all i} \qquad (6)$$

and

$$\sum_i a_i p_{ij} = a'_j \text{ for all j} \qquad (7)$$

where $a_i$ are elements of $a_0$; $a'_j$ are elements

[15] E. Sibley, "Some Demographic Clues to Stratification," *American Sociological Review,* 7 (June, 1942), pp. 322–330; Joseph A. Kahl, *The American Class Structure,* New York: Rinehart, 1957, Chap. 9; Matras, "Differential Fertility . . . ," *op. cit.;* S. Yasuda, "A Methodological Inquiry into Social Mobility," *American Sociological Review,* 29 (February, 1964); and Duncan "Methodological Issues . . . ," *op. cit.*

[16] Matras, "Differential Fertility . . . ," *op. cit.*

[17] Gösta Carlsson, *Social Mobility and Class Structure,* Lund, Sweden: Gleerup, 1958, Chap. 7.

[18] H. C. White, "Cause and Effect in Social Mobility Tables," *Behavioral Science,* 8, (January, 1963), pp. 14–27; See also L. A. Goodman, "On the Statistical Analysis of Mobility Tables," *American Journal of Sociology,* 70 (March, 1965), pp. 564–585.

of $a_1$; and $p_{ij}$ are transition probabilities of M.

These simultaneous equations and inequalities, (5), (6) and (7) can be solved by, say, an appropriate computer routine, to yield upper and lower bounds for the entries, $p_{ij}$, of the social mobility matrix, M. Alternatively, if the social or occupational categories are ranked, the stronger assumption that mobility rates or transition probabilities are smaller the greater the difference between the ranks of the categories in question, i.e., the greater the distance from the major diagonal in the matrix, M, imposes the conditions

$$p_{ij} \geqq p_{i'j'} \text{ for all } i, j, i', j' \text{ such that } |i-j| \leqq |i'-j'| \quad (8)$$

Again, the simultaneous equations and inequalities, (6), (7) and (8) can be solved to yield bounds for the $p_{ij}$. Other kinds of conditions, e.g., "maximum mean distance", "minimum mean distance", "maximum" or "minimum" volumes of movement conditions, could be imposed upon the mobility process matrix, M, having one or another type of linear programming solution or bound set of solutions.

Considering now the problem of plotting, analysing, or predicting the changes in social structure over time associated with a given social mobility process, M, operating upon a specified initial social structure, $a_0$, it may be recalled that, in equation (1), the social structure, $a_0$, and the given set of mobility rates or transition probabilities, M, completely and uniquely specify the subsequent social structure, $a_1$. Similarly, the structure, $a_1$, and a mobility process occurring in the next interval, represented, say, by a matrix $M_1$, completely and uniquely specify the next new subsequent structure, $a_2$. In general, the relationship between social structure and social mobility processes occurring over many, say, k, successive intervals is given by equation (2)

$$a_0 M_1 M_2 M_3 \cdots M_{k-1} M_k = a_k$$

What is in question in this type of problem is the nature of the relationship between the successive matrices, $M_1$, $M_2$, ... $M_{k-1}$, $M_k$, in equation (2).

The now-familiar Markov-chain assumption is that the social mobility matrix, M, does not change over successive time intervals, i.e., $M_1 = M_2 = M_3 = \ldots = M_k = M$, whence

$$a_0 M^k = a_k \quad (9)$$

where $M^k$ is the original transition probability matrix raised to the k-th power. The usual assumption of the model of equation (9) that the movements in any given interval are independent of those of the preceding or of any previous interval can be varied in at least two ways, viz., by partitioning the population into mobile and nonmobile or "mover" and "stayer" parts,[19] or by use of a second- or higher-order Markov chain model instead of the usual first-order model.[20]

It would appear no less reasonable however to assume that, in any given interval, the social mobility process *is* dependent upon the social structure at the beginning of the interval,[21] i.e., that

$$M_v = g(a_{v-1}) \quad (10)$$

If the matrix-valued function, g, is fixed over all intervals, $\nu = 1, 2, \ldots, k$, then the initial social structure, $a_0$, and the relationship between the social mobility process and the social structure, g, completely and uniquely determine the social structure at any subsequent time, i.e.,

$$a_k = a_{k-1} M_k = a_{k-1}\{g(a_{k-1})\} = a_0\{g^{(k)}(a_0)\} \quad (11)$$

where $g^{(1)}(a_0) = g(a_0) M_1$;
$g^{(2)}(a_0) = g(a_0) g(a_0 g(a_0)) = M_1 M_2$;
$\ldots$;
$g^{(k)}(a_0) = M_1 M_2 M_3 \ldots M_{k-1} M_k$.

Theoretical or empirical considerations may be brought to bear in determining the nature or form of the relationship, "g." For example, a "takeoff" or "stages of development" theory would imply, say, more or less stable patterns of mobility in successive intervals until some particular proportion or combination of proportions of the labor force in the specific occupational groups or industrial sectors is reached. Afterward some new mobility patterns might operate until a dif-

[19] Blumen, Kogan, and McCarthy, *op. cit.;* and Goodman, "Statistical Methods for the Mover-Stayer Model," *op. cit.*

[20] L. A. Goodman, "Statistical Methods for Analyzing Processes of Change," *American Journal of Sociology,* 58, 1962.

[21] Duncan "Methodological Issues . . . ," *op. cit.*

ferent "stage" or specific type of structure is reached, and so forth. Alternatively, a theory of changing social structure and, say, the structure of consumption and demand might be invoked to relate industrial and occupational mobility to the social structure. An empirically-based procedure might associate mobility processes, $M_a$, $M_b$, $M_c$, $\ldots$, in different societies, with the initial social structures in association with which they are observed, $\alpha_a$, $\alpha_b$, $\alpha_c$, $\ldots$ etc., and set

$$g(\alpha_a)=M_a;\ g(\alpha_b)=M_b;\ \text{etc.}$$

For any given society, then, prediction of the mobility process in the $\nu$-th interval, $M_\nu$, would entail comparing the social structure at time $t=\nu-1$, i.e., comparing $\alpha_{\nu-1}$ with the model structures, $\alpha_a$, $\alpha_b$, $\alpha_c$, $\ldots$ etc. and choosing the corresponding mobility process, $M_u=g(\alpha_u)$ such that $(\alpha_{\nu-1}-\alpha_u)$, the difference between the actual $(\nu-1)$-st and the available model structures, is minimized.

## A MODEL OF SOCIAL MOBILITY, POPULATION GROWTH AND CHANGING SOCIAL STRUCTURE

In the usual matrix equation representation of social mobility and changing social structure [equation (1)] it seems clear that, *if* the $\alpha_0$ and $\alpha_1$ represent, say, the same closed labor force at time 0 and at time 1 respectively, *or if* $\alpha_1$ represents the social structure of a sample of sons and $\alpha_0$ represents the social structure of their fathers, *then* the equation $\alpha_0 M=\alpha_1$ is a realistic representation of a social structure and the manner in which the change in social structure is related to a social mobility process represented by the matrix, M. *Otherwise,* the equation is very considerably short of being a realistic portrayal of actual societies or populations. O. D. Duncan has shown that, if $\gamma_0$=the social structure of a society or population at time $t=0$, and $\gamma_1$=the social structure of the same society or population at time $t=1$ then, although we would like to formulate some transformation, T, such that

$$\gamma_0 T=\gamma_1,$$

in fact none of the models of the form $\alpha_0 M=\alpha_1$ is an adequate or realistic approximation of $\gamma_0 T=\gamma_1$. For if we are dealing with, say, intergenerational mobility, and if we take $\alpha_1=\gamma_1$ then ordinarily $\alpha_0\neq\gamma_0$; or if we take $\alpha_0=\gamma_0$, then generally $\alpha_1\neq\gamma_1$; and in any event $M\neq T$. Duncan points out that the change from $\gamma_0$ to $\gamma_1$ takes place by differential population growth *and* by social mobility jointly, i.e., that T is a socio-demographic process rather than strictly a social mobility process.[22] What is required, then, is a representation of T, the socio-demographic process taking the society from $\gamma_0$ to $\gamma_1$, when $\gamma_0$ and $\gamma_1$ are actual social structures rather than samples of closed sub-populations.

A representation of such a socio-demographic process is suggested by the linear model employed by N. Keyfitz for carrying out population projections using the electronic computer.[23] Keyfitz's model projects the female population, beginning first with an initial population classified by age groups, say $W_0=(w_1, w_2, \ldots, w_i, \ldots, w_n)$. The probabilities that a woman in the i-th age group survives, say s(i), and that a woman in the i-th age group bears a daughter, say f(i), in an interval are built into a projection matrix, say

$$P=\begin{pmatrix}
f_{(1)} & s_{(1)} & 0 & 0 & \cdot & 0 & \cdot & 0\\
f_{(2)} & 0 & s_{(2)} & 0 & \cdot & 0 & \cdot & 0\\
f_{(3)} & 0 & 0 & s_{(3)} & \cdot & 0 & \cdot & 0\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
f_{(i)} & 0 & \cdot & \cdot & \cdot & s_{(i)} & \cdot & 0\\
\cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0\\
\cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0\\
f_{(n-1)} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & s_{(n-1)}\\
f_{(n)} & 0 & 0 & 0 & 0 & 0 & 0 & s_{(n)}
\end{pmatrix}$$

and the size and age distribution of the female population at the end of the interval are given,

$$W_0 P=W_1 \qquad (12)$$

We may adapt Keyfitz's projection model to a male population classified by age *and* by social or occupational class or category, by allowing them, in a given interval, to survive or not, to sire a son or not, *and* to change social or occupational class or not.

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[22] *Ibid.*

[23] N. Keyfitz, "Matrix Multiplication as a Technique of Population Analysis," *Milbank Memorial Fund Quarterly,* 42 (October, 1964), pp. 68–84; N. Keyfitz, "Utilisation des machines électroniques pour les calculs demographiques," *Population,* 19 (August–September, 1964), pp. 673–682; and Nathan Keyfitz and Edmond M. Murphy, *Comparative Demographic Computations,* Chicago: Population Research and Training Center, University of Chicago, 1964.

Let $w_{ij}$ = the number of males in the i-th age group and in the j-th social or occupational class category at time $t=0$;

$s_{ijj'}$ = the rate at which males in the i-th age group and j-th social class survive and are in the j'-th social class at the end of an interval, say at $t=1$;

$f_{ij}$ = the rate at which males in the i-th age group and in the j-th social class at the beginning of the interval sire sons who survive to the end of the interval.

If we have, say, n age groups and m social classes, then the socio-demographic structure of the population at time $t=0$ can be represented, $\gamma_0$, by a vector or a (1 x nm) matrix,

$$\gamma_0 = (w_{11}, \ldots, w_{1m}, w_{21}, \ldots, w_{2m}, \ldots, w_{ij}, \ldots, w_{n1}, \ldots, w_{nm})$$

and the socio-demographic process transforming the social structure, T, may be represented by an nm x nm matrix of the form,

$$T = \begin{pmatrix}
f_{11} \ldots 0 \ldots 0 & s_{111} \ldots s_{11j'} \ldots s_{11m} & \cdot & 0 \quad 0 \quad 0 & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 \ldots f_{1j} \ldots 0 & s_{1j1} \ldots s_{1jj'} \ldots s_{1jm} & \cdot & 0 \quad 0 \quad 0 & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 \ldots 0 \ldots f_{1m} & s_{1m1} \ldots s_{1mj'} \ldots s_{1mm} & \cdot & 0 \quad 0 \quad 0 & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
f_{i1} \ldots 0 \ldots 0 & 0 \ldots 0 \ldots 0 & \cdot & s_{i11} \ldots s_{i1j'} \ldots s_{i1m} & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
0 \ldots f_{ij} \ldots 0 & 0 \ldots 0 \ldots 0 & \cdot & s_{ij1} \ldots s_{ijj'} \ldots s_{ijm} & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 \ldots 0 \ldots f_{im} & 0 \ldots 0 \ldots 0 & \cdot & s_{im1} \ldots s_{imj'} \ldots s_{imm} & \cdot & 0 \quad 0 \quad 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
f_{n1} \ldots 0 \ldots 0 & 0 \ldots 0 \ldots 0 & \cdot & 0 \quad 0 \quad 0 & \cdot & s_{n11} \ldots s_{nij'} \ldots s_{n1m} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 \ldots f_{nj} \ldots 0 & 0 \ldots 0 \ldots 0 & \cdot & 0 \quad 0 \quad 0 & \cdot & s_{nj1} \ldots s_{njj'} \ldots s_{njm} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 \ldots 0 \ldots f_{nm} & 0 \ldots 0 \ldots 0 & \cdot & 0 \quad 0 \quad 0 & \cdot & s_{nm1} \ldots s_{nmj'} \ldots s_{nmm}
\end{pmatrix}$$

whence the equation

$$\gamma_0 T = \gamma_1 \qquad (13)$$

describes the change in social structure, $\gamma_0$ to $\gamma_1$, as affected by differential fertility and mortality and by social mobility jointly.

Practically it would seem necessary to make some simplifying assumptions concerning timing of births relative to mobility. Similarly some formula for connecting the mobility of fathers to that of their dependent sons must be derived theoretically or empirically. Finally, assumptions about differential ages at entrance into the labor force, or at passage of sons from ascribed (fathers') status or achieved (own) status must be derived and expressed with the appropriate $S_{ijj'}$.

Application of this model to actual data will surely demand variations and improvisations. If, as contended in the previous sections above, the linear models have genuinely contributed to our understanding of the relationship between social mobility and social structure, then the attempt to apply such a complex demographic-mobility model comparatively would appear to hold some promise for adding to our insights concerning such relationships.